Metadata of the chapter that will be visualized online

Chapter Title	Combinatorial Auctions	
Copyright Year	2012	
Copyright Holder	Springer Science+Business Media, LLC	
Corresponding Author	Family Name	**Hoffman**
	Particle	
	Given Name	**Karla**
	Suffix	
	Division/Department	Department of Systems Engineering and Operations Research
	Organization/University	George Mason University
	Street	4400 University Drive
	City	Fairfax
	Postcode	22030-4444
	State	VA
	Country	USA
	Email	khoffman@gmu.edu

C

Combinatorial Auctions

Karla Hoffman
Department of Systems Engineering and Operations Research, George Mason University, Fairfax, VA, USA

Galley Proof

Introduction

The advent of the Internet has led to the creation of global marketplaces in which sales of everything from low-cost used merchandise to billion dollar government procurements are conducted through auctions. This article concentrates on designs where many items are auctioned simultaneously and where bidders have the flexibility to combine the goods into packages. The discussion (1) highlights alternative combinatorial auction designs and provides the reader with multiple references to resources that describe more fully the underlying theory of these designs., and (2) describes the mechanisms used to evaluate the efficacy of such approaches in terms of their efficiency, equity, and cognitive complexity, and presents some examples of the use of combinatorial auctions for high-value government lease rights, as well as the use of such auctions for supply-chain procurement. These auctions require knowledge of both game theory and combinatorial optimization.

General Concepts

Governments throughout the world use auctions to lease the right to explore and extract minerals, fuel, and lumber on government properties, to use the airwaves for mobile or broadcast communications, or to control emissions through cap and trade regulations. In addition, the use of business-to-business auctions (often called supply chain auctions) has become a billion-dollar industry. In each of these cases, the need to be able to bundle buys and sells has resulted in new auction theory and designs that enable the simultaneous selling or buying of items using mechanisms that allow participants to indicate their value for the entire package which may have a greater value than the sum of the items within that package. In addition, such auction designs allow users to specify quantity discounts, to indicate budget constraints on the total procurement, and to define other goals of the auction, e.g. social welfare goals in a government auction. These auction designs are computationally more complex for all participants and require languages that allow bidders to express their willingness to participate at a given price for a collection of objects. Such auctions have been termed combinatorial auctions. There are many books that describe the history of auctions, auction theory and its relationship to game theory, and others that are focused exclusively on combinatorial auction designs. For further reading on the subject, see: McMillan (2002) on the history of markets, Krishna (2002) on auction theory, Steiglitz (2007) on the success and pitfalls of EBAY auctions, Klemperer (2008) on auction theory and practice, and Milgrom (2004) and Cramton et al. (2005) on combinatorial auctions. In this review, only the major topics of the field are described, but multiple references are provided for further reading. Au1

S. Gass, M. Fu (eds.), *Encyclopedia of Operations Research and Management Science*,
DOI 10.1007/978-1-4419-1153-7, © Springer Science+Business Media, LLC 2012

Comp. by: MANIKANDAN R Stage: Galleys Chapter No.: 1139 Title Name: EORMS
Date:11/8/12 Time:06:48:43 Page Number: 2

In what follows, one-sided auctions are considered and are restricted to the case where there is a single seller and multiple buyers (two-sided auctions are often referred to as exchanges, see Milgrom (2007), Parkes et al. (2001), and Hoffman and Menon (2010) on exchange designs). Since the multiple-sellers/single-buyer case and the multiple-buyers/single-seller case are symmetric, the discussion emphasizes the latter, but all results follow for either case. The concentration is on auction designs where that there are multiple items being sold. For at least some of the buyers, a collection of items must be procured to have a viable business plan; consideration is given only to auction designs that allow the packaging of collections of items. Such designs can provide greater efficiency, as well as greater revenue to the seller than the sequential selling of items individually. These designs are sufficiently general to allow bidders to express a value on a package where the collection of items may have a value greater than the individual items (i.e. the goods are complements), as well as on a package where a buyer can express a quantity discount for buying more of the good (i.e. the goods are substitutes).

Why are auctions such a popular mechanism for buying and selling valuable objects? With the advent of the Internet, auctions are capable of reaching many more possible participants. Here, the potential buyers wish to determine the minimum price that they must pay given that they must compete with others for the ownership of a good or collection of goods. From the seller's perspective, submitting goods to an auction may increase the number of buyers, thereby increasing the potential for competitive bidding and higher selling prices. Thus, an auction is a mechanism to determine the market-based price, since the bidders set the price through the competition among the bids. This mechanism is dynamic and reacts to changes in market conditions. The determination of selling prices by an auction is perceived as fairer than if the price were set by bilateral negotiations because all buyers must adhere to the same set of rules. Most importantly, if the rules are well designed, the result will have the goods allocated to the entity that values them the most.

The two basic classes of auctions are described next: (1) sealed bid auctions whereby there is only a single opportunity to provide bids to the auction, and (2) multi-round auctions where bids are taken over a period of time and any high bid can be overtaken whenever a new bid is received that increases the overall revenue to the seller.

Sealed Bid Auctions

One common auction mechanism is the first-price (sealed bid) auction. In this design, all bidders submit their bids by a specified date. The bids are examined simultaneously and the auctioneer determines the set of bidders that maximizes the revenue to the seller. The optimization problem that determines a collection of package bids that do not often overlap and produce the maximum revenue is known as the Winner Determination Problem (WDP). Mathematically, the problem can be stated as follows:

$$
\begin{aligned}
&WDP_{OR} : Max \sum_{b=1}^{\#Bids} BidAmount_b x_b \\
&subject\ to: \\
&Ax \leq 1
\end{aligned}
\tag{1}
$$

$$
x \in \{0, 1\} \tag{2}
$$

where x_b is a zero–one variable which indicates whether bid b loses or wins, respectively. A is an n x m matrix with m rows, one for each item being auctioned. Each of the n columns represents a bid where there is a one in a given row if the item is included in the bid and zero otherwise. Constraint set (1) specifies that each item can be assigned at most once. Set (1) constraints are equations when the seller chooses to put a minimum price on each item and is unwilling to sell any item below that price. In this case, there is a set of m bids each with only a single item in the package and a bid price at a price slightly below the minimum opening bid price. In this way, the seller will keep the item rather than allow it to be won by a bidder at less than the opening bid price.

In this formulation of the WDP, the bidder can win any combination of bids, as long as each item is awarded only once; this is referred to as the "OR" language. The problem with this language is that it creates a type of exposure problem, that of winning more than the bidder can afford. When multiple bids of a single bidder can be winning, it is incumbent on the software to highlight the maximum exposure to the

Galley Proof

Comp. by: MANIKANDAN R Stage: Galleys Chapter No.: 1139 Title Name: EORMS
Date:11/8/12 Time:06:48:44 Page Number: 3

bidder. This calculation requires that a combinatorial optimization problem be solved for each bidder that calculates the dollar exposure, creating new computational issues for the auctioneer and may result in packages that are not best for the bidder.

The most natural alternative to this "OR" language is the "XOR" language. In this case, the user supplies every possible combination of bids of interest along with a maximum bid price that she is willing to pay for that package. This language removes the dollar exposure problem, since the maximum number of bids that a bidder can possibly pay is the highest bid amount of any of its bids. The problem with the XOR language is that it places a new burden on the bidder: the bidder is forced to enumerate all possible combinations of packages of interest and their associated values. Clearly, as the number of items in an auction increase, the number of possible bids goes up exponentially. When the XOR bidding language is used the Winner Determination Problem (WDP_{XOR}) becomes:

$$
\begin{aligned}
WDP_{xor} : Max \quad & \sum_{b=1}^{\#Bids} BidAmount_b x_b \\
& subject\ to : \\
& x = 1
\end{aligned}
\tag{3}
$$

$$
\sum_{b \in S_B} x_b \leq 1 \text{ for each bidder B} \tag{4}
$$

$$
x_b \quad \in \{0, 1\} \tag{5}
$$

Where S_B is the set of bids of bidder B, and constraint set (4) specifies that at most one of these bids can be in the winning set.

Fujishima et al. (1999) proposed a generalization of the OR language that does not require the enumeration of all possible combinations. They label this language OR*. Here, each bidder is supplied dummy items (these items have no intrinsic value to any of the participants). When a bidder places the same dummy item into multiple packages, it tells the auctioneer that the bidder wishes to win at most one of these collections of packages. This language is fully expressive, as long as bidders are supplied sufficient dummy items. This language is also relatively simple for bidders to understand and use, as was shown in a Sears Corporation supply-chain transportation auction. In that auction, all bids were treated as "OR" bids by the system. Some bidders cleverly chose a relatively cheap item to place in multiple bids thereby making these bids mutually exclusive, Ledyard et al. (2002). There have been a number of alternative bidding languages that have been proposed; see Fujishima et al. (1999), Nisan (2000), Boutilier and Hoos (2001), and Boutelier et al. (2002) for descriptions of alternative languages.

One serious flaw in a first-price sealed-bid design is that the bidder can experience what is referred to as the winner's curse, i.e., the winning bidder may pay more than was necessary to win since the second highest bid price was far less than the winning bid amount. For this reason, sealed-bid first price auctions encourage bidders to shave some amount off of the bid price. From a game-theoretic perspective, one wants an auction design that encourages straight-forward honest bidding.

An alternative that overcomes this problem is the second price (sealed bid) auction whereby the bidder that has submitted the highest bid is awarded the object (package), but the bidder pays only slightly more (or the same amount) as that bid by the second-highest bidder. In second price auctions with statistically independent private valuations, each bidder has a dominant strategy to bid exactly his valuation. The second price auction also is often called a Vickrey auction (1961).

In a second-price auction, one solves the same winner determination problem as one does for the first-price sealed-bid case, but the winners do not necessarily pay what they bid. Instead, one determines the marginal value to the seller of having this bidder participate in the auction. To do this, for each winning bidder, one calculates the revenue that the seller would receive when that bidder participates in the auction and when that bidder does not, i.e. when none of the bids of this bidder are in the winner determination problem. The difference in the two objective function values is known as the Vickrey-Clarke-Groves discount, named after the three authors, Vickrey (1961), Clarke (1971), and Groves (1973). Each of these authors wrote separate papers producing certain attributes that this auction design has as it relates to incentivizing bidders to reveal their truth value of the goods demanded, and the bidder pays the bid price minus the discount. When

Galley Proof

Comp. by: MANIKANDAN R Stage: Galleys Chapter No.: 1139 Title Name: EORMS
Date:11/8/12 Time:06:48:44 Page Number: 4

winners pay this amount, the auction is known as the Vickrey-Clarke-Groves (VCG) Mechanism.

Although it can be shown that the VCG mechanism encourages truthful bidding, it is almost never used in practice. For a complete list of reasons for it being impractical, see Ausubel and Milgrom (2006) and Rothkopf (2007). In essence, the prices provided by this mechanism may be very low. Worse yet, when items have complementary values, i.e. the package is worth more to the bidder than the sum of the values of the individual items, the outcome may price the items so low that there is a coalition of bidders that would prefer to renege on the auction and negotiate privately with the seller, and the seller may respond by reneging on the sale since both the seller and the coalition of buyers will be better off. Ausubel and Milgrom (2002) argue that prices should be set high enough so that no such coalitions exist. In game theoretic terms, the prices are set such that the outcome is in the core of a coalitional game. These authors introduced an auction design known as the ascending proxy auction in which the bidders provide all bids as if in a sealed-bid auction. Each bidder is provided with a proxy that bids for the bidder in a straightforward manner during an ascending auction. The proxy only announces bids to the auctioneer that maximize the bidder's profit, (i.e. bid price minus announced price) in any given round. The auction continues as an ascending package-bidding auction until, in some round, there are no new bids. Thus, the auction simulates, through proxy bidders, an ascending auction where the increment in each round is infinitesimally small and each bidder, through the use of its proxy, bids in a straight-forward manner. This auction design is very similar to the iBundle design of Parkes and Ungar (2000).

Hoffman et al. (2006) provide a computational approach toward speeding up the calculations associated with this proxy auction design, and Day and Raghavan (2007) provide an elegant mechanism to obtain minimal core prices directly. The direct mechanism of Day and Raghavan sequentially solves winner determination problems to determine losing coalitions that could supply more revenue to the seller at the current prices. When the solution to this optimization problem yields revenue greater than what the VCG mechanism would provide, the prices of the winning bid set are raised so that the total price paid by winning bidders is equal to this new revenue. To determine these new prices, one must be sure that any winning bidder that forms part of this blocking coalition does not have its price raised from its prior price since it would not be willing to join a coalition if it were to lose revenue relative to its prior offer by the seller. The algorithm is an iterative cutting plane algorithm that forces the prices higher at each iteration until one can find no coalition that can increase revenue to the seller. Therefore, the algorithm finds prices for each winning bidder that are in the core. Since there may be many such minimum core prices, Day and Milgrom (2008) suggest that, in order to encourage sincere bidding, one choose the minimum core prices that are closest in Euclidean distance from the VCG prices. Alternatively, Erdil et al. (2009) argue for a different set of minimum core prices that are based "on a class of 'reference rules' in which bidders' payments are, roughly speaking, determined independently of their own bids as far as possible."

These core-selecting second-price sealed-bid mechanisms have the following properties: They are in the core, they eliminate the exposure problem, and they encourage bidders to bid sincerely. As with all sealed-bid auctions, they make collusion and punishment for not adhering to tacit agreements extremely difficult.

There are, however, negatives associated with this auction, as well as for all sealed-bid auction designs, in that it puts a significant burden on the bidders. Each bidder needs to assess, for every possible combination of items, whether it is a package of interest and then, for all such packages, determine the maximum it is willing to pay. In addition, such mechanisms do not provide any information about how the packages submitted might fit with packages submitted by other bidders. To overcome these problems, a number of authors have suggested simultaneous ascending combinatorial auction designs that allow users price information during the auction.

Multi-round Auctions

Often the value of the good or package of goods being auctioned is not completely known and/or private. Instead, there is a common component to the bid value, that is, the value of the item is not independent of the other bidders, but rather there is a common

Galley Proof

Comp. by: MANIKANDAN R Stage: Galleys Chapter No.: 1139 Title Name: EORMS
Date:11/8/12 Time:06:48:44 Page Number: 5

underlying value as well. In such situations, each agent has partial information about the value. Many high-stakes auctions, such as government auctions for spectrum, oil exploration, and land use, fall into this class. In the case of package-bidding auctions, when there is a common component and bidders want to assess how much others are willing to pay for that item or package of items, the auction is usually an ascending auction with multiple rounds. A round consists of a given time period where bidders have the opportunity to submit new bids. When the round ends, all bids are collected and the winner determination problem is solved. This optimization problem determines the packages that provide the seller with the maximum revenue. The bids that are in the winning set are labeled "provisionally winning," i.e. they would be winning if the auction ended in this round. Thus, in an ascending combinatorial auction, all items are sold simultaneously and a bidder can bid on any collection of items in a given round. To overcome the current set of provisionally winning bids, a bidder must submit a bid that increases the total revenue to the seller.

There are a number of design question that must be answered to have a complete combinatorial auction design:

1. How does the auction end?
2. Must bidders participate in every round?
3. Are bids from previous rounds part of the bids considered by the winner determination problem?
4. How are the prices set in each round?
5. What do bidders know about the bids of other bidders?
6. What other rules might be necessary to ensure that collusion is avoided, to make reneging costly, and to encourage bidders to act truthfully?

Of importance is how to assure that the auction ends in a reasonable period of time and that price discovery (the main reason for a multi-round auction) is accomplished. Most package-bidding auctions have discrete time periods, called rounds, and in each round, the auctioneer provides a price to the user that is the minimum price that the bidder must supply in order to place a new bid. One can choose either a fixed stopping rule or a stopping rule that is determined dynamically. A fixed time stopping rule specifies that the auction will end at a given time. With a fixed stopping time, bidders are encouraged to not provide any bids until the very last seconds of the auction, called sniping. The purpose of sniping is to give other bidders no chance of responding to an offer. In this way, a bidder can acquire price information from other bidders but does not reciprocate, since throughout most of the auction, the bidder is silent. If all bidders chose to snipe and provide no bids until the end of the auction, the auction essentially becomes a first-price sealed-bid auction. To overcome the problem of sniping and to encourage price discovery, most package bidding auctions use an alternative stopping criteria whereby the auction ends when no new bids are presented within a round.

Often, for high-stakes multi-round auctions, there are also activity rules that require a bidder to bid in a consistent way throughout the auction. Activity rules force bidders to maintain a minimum level of bidding activity to preserve their eligibility to bid in the future. Thus, a bidder desiring a large quantity at the end of the auction (when prices are high) must bid for a large quantity early in the auction (when prices are low). If the bidder cannot afford to bid on a sufficient number of items to maintain current eligibility, then eligibility will be reduced so that it is consistent with current bidding. Once eligibility is decreased, it can never be increased. As the auction progresses, the activity requirement increases, reducing a bidder's flexibility. The lower activity requirement early in the auction gives the bidder greater flexibility in shifting among packages early on when there is the most uncertainty about what will be obtainable. Precisely how the activity and eligibility rules are set matters and must be depend upon the type of auction – the value of the items being auctioned, the projected length of the auction, the number of participants, etc. In many high-stakes auctions, such as spectrum or electricity, these activity rules have proven highly successful, Klemperer (2002), McMillan (2002), and Milgrom (2004).

In an ascending multi-round auction design, the auctioneer must provide information about the current value of each package. This information is used for two related purposes: (1) to specify the minimum bid for each item or package in the next round and (2) to provide valuation information to bidders so that they can determine what might be required for a bid to be winning in a subsequent round. While pricing information is easy to ascertain in single item auctions or in simultaneous multi-round auctions without package bidding, (i.e. where bids can

Galley Proof

Comp. by: MANIKANDAN R Stage: Galleys Chapter No.: 1139 Title Name: EORMS
Date:11/8/12 Time:06:48:44 Page Number: 6

be placed on only single items), pricing information for combinatorial auctions is not well defined. Bidders provide only aggregate package prices without providing the information about how each of the individual components that made up the bundle contributes to the overall price. Attempting to disaggregate these bundles into single item prices unambiguously is not possible. Also, since there are many ways that some bundle might partner with other packages to create a winning set, determining the minimal cost partnering for a given package by a given bidder is a complex problem.

To further complicate the pricing issue, bidders may view certain items as substitutes and other items as complements. In the case where items are substitutes, bidders are likely to express sub-additive values for their packages. That is, the value of a package of items is less than or equal to the sum of the values of the items that make up the package. In the complementary case, bidders are likely to express super-additive values for packages. In this case, the value of a package of items is greater than or equal to the sum of the values of the items that make up the package. When items can be both substitutes and complements for bidders, providing unambiguous, complete and accurate price information is an unsolved problem. The non-convex nature of the problem means that the linear prices (i.e. the sum of a package is equal to the sum of the individual items that make up the package) that can be obtained from dual prices from the linear relaxation of the WDP problem will overestimate the true values of the items. In most auctions, one adjusts the dual prices so that the prices are modified so that when one sums the items in each of the winning packages, the prices on those packages exactly equal the prices bid by the provisionally winning bidders (i.e. the winners at the end of the current round). Rassenti et al. (1982) terms these prices pseudo-dual prices. (For theoretical issues with duals associated with non-convex problems see Wolsey(1981), and for non-anonymous non-linear prices see deVries and Vohra (2003) and Bikhchandani and Ostroy (2001).)

Although linear pricing cannot accommodate all aspects of the pricing associated with the non-linear, non-convex, winner determination problem, there are still good reasons for considering its use for determining future bid requirements. First, even perfect pricing is only correct when all other aspects of the problem remain fixed, i.e. when bid amounts remain the same on all other bids and when no new bids are submitted. Second, a dual price associated with a given constraint is only correct when one changes this single restriction (the right-hand-side of the associated constraint) by a very small amount. In the case of combinatorial auctions, the item is either won or it is not. Changes to a constraint would either remove the item entirely from consideration or create a second identical item. Thus, even non-linear, non-anonymous pricing has serious limitations in the context of the winner determination problem since the removal of a single item from the auction (e.g. the removal of the New York City market from consideration in a nationwide spectrum auction) may change the willingness of bidders to participate.

Finally, in an ascending bid auction, bidders need pricing information that is easy to use and understand, and is perceived to be fair. In this situation, easy to use means that bidders can quickly compute the price of any package, whether or not it had been previously bid. Often, bidders want to know what it would take for such a bid to be competitive, i.e. have some possibility of winning in the next round. Bidders may also perceive such prices to be fair since all bidders must act on the same information. Linear prices are likely to move the auction along and deter such gaming strategies as parking (parking is an approach whereby the bidder bids on packages that currently have very low prices knowing that these packages have a very low probability of winning). Bidding on such low-priced packages allows a bidder to maintain eligibility (by maintaining activity), while hiding interest in the packages that are really desired until later in the auction). Thus, virtually all ascending combinatorial auctions use pseudo-dual pricing. For more on alternative pricing within this general framework and the testing thereof, see (Dunford et al. (2003), Bichler et al. (2009) and Brunner et al. (2011).

In 1999, DeMartini et al. proposed an auction design labeled The Resource Allocation Design or RAD where the WDP is solved each round and all losing bidders can only bid on packages where the package price is the sum of the pseudo-dual prices plus some increment (as announced by the auctioneer). There is no activity rule for this auction design. In 2002, the Federal Communications Commission (FCC) announced a similar package bidding design but proposed refinements to the pseudo-price calculations that attempts to limit

Galley Proof

Comp. by: MANIKANDAN R Stage: Galleys Chapter No.: 1139 Title Name: EORMS
Date:11/8/12 Time:06:48:45 Page Number: 7

fluctuations (both positive and negative) in prices. A related design was proposed by Bichler et al. (2009) and is called the Approximate Linear Pricing Scheme (ALPS). It also uses similar rules but chooses the ask price to better balance prices across items. Note that all of these pricing procedures allow prices to both increase and decrease depending upon the packages that are in the winning set. In virtually all of these designs, any bid submitted in any round is considered active throughout the auction. This rule works well with the XOR language since only one bid of a bidder can be in an optimal set and bidders should be willing to win bids placed in early rounds of the auction, when prices were low. This rule forces bidders to provide sincere bids throughout the auction.

A very different ascending package bidding design was proposed by Porter et al. (2003). It is called the combinatorial clock auction. In this design, the auctioneer provides prices for each unique good (if there are multiple identical items, then the bidder indicates that number of units of that item they desire) based solely on whether there is more demand for the item than for supply; no WDP problem is solved. There is no concept of a provisionally winning bidder. Instead, prices increase whenever demand for a given item is greater than supply. Bidders indicate the single package bid that is best given the per-unit prices announced by the auctioneer. All bidders must rebid on any item that they wish to procure in each round. The only information provided to bidders at the end of each round is the quantity demanded for each item and the price for the next round. As long as demand exceeds supply for at least one item, the price is increased for those items with excess demand. If there are no new bids in a round and supply equals demand, then the auction ends. However, it may happen that when there are no new bids, demand has been reduced to below supply. If this occurs, a WDP is solved using all bids from all rounds. If the computed prices do not displace any bids from the last round, then the auction ends. Otherwise, the auction resumes with the prices determined by using the pseudo-prices calculated from the WDP. Thus, for most rounds, the computation has been drastically reduced to merely increasing prices by a given increment. Only, when demand has dropped below supply is the WDP solved.

Other approaches are the auction designs that simplify the problem by only allowing a few pre-defined packages (Harstad et al. 1998) for which the WDP is polynomially solvable. This idea of only allowing a certain pre-determined set of packages (called hierarchical packages, Goeree and Holt 2010) was used in the 2009 FCC auction for broadband spectrum that brought over $19B into the U.S. Treasury. In that design, all bids were additive (the OR language applied) and the WDP was solved in linear time. When it is possible, in advance, to understand the needs of the bidders and when the packages most desired can be represented in a hierarchical fashion, then one obtains an auction design that is both simpler and quite efficient. However, if the demand for packages does not take on this hierarchical structure, then imposing such structure on the problem for the sake of computability will likely lead to less efficient outcomes.

Hybrid Designs

Ausubel et al. (2006) have argued for a hybrid design that reduces the computational burden on both the bidder and the auctioneer. Here, one first uses a combinatorial clock design followed by a last round second-price sealed-bid approach. The combinatorial clock is similar to that proposed by Porter et al. (2003) with the further enhancement that bidders who find the increment too high are able to place a bid at a price between the old price and the new price that indicates the maximum amount the bidder is willing to pay for that combination of items. In this way, the efficiency loss due to increment size is lessened. This phase of the auction ends when demand is less than or equal to supply or when demand on most items has trailed off. When demand does not exactly equal supply on all items, a sealed-bid phase is initiated. Here, the ascending proxy auction of Ausubel and Milgrom (2001) is imposed. When these two auction designs are merged, one must be careful that the activity rules work well for both phases of the auction. One wants tight activity rules in the ascending phase of the auction to ensure that the bidders are forced to bid sincerely. However, these rules may need to be relaxed or altered during the final sealed-bid phase or a straightforward bidder may be precluded from providing all of the packages that bidder values during the sealed-bid round. Also, theory dictates that in order to guaranteed an efficient outcome, losing

Galley Proof

Comp. by: MANIKANDAN R Stage: Galleys Chapter No.: 1139 Title Name: EORMS
Date:11/8/12 Time:06:48:45 Page Number: 8

bidders (i.e. bidders who dropped out prior to the final phase) must also provide all of the bids that they value in the final phase. Thus, although this hybrid auction is promising in that it is likely to speed up combinatorial auctions, research is still necessary to better understand how the rules of these two disparate auctions should be set so that they mesh well. For more on testing of this design, see Bichler et al. (2011).

Complexity of Combinatorial Auctions

As the previous discussion illustrates, most combinatorial auction designs requires considerable computation and most of the computational burden falls to the auctioneer. This seems appropriate since the auctioneer wants an auction that allows much participation; bidders should not be required to understand combinatorial optimization in order to participate. In terms of these computations, commercial software, such as CPLEX, GUROBI, or XPRESS have shown their ability to solve such problems in reasonable times (less than 30 minutes). Thus, although there is much in the literature that argues against combinatorial auctions because of the computational burden, the optimization software has proven up to be capable of handling the problems that are currently being considered applicable for this type of auction. For more on the computational issues in computing winner determination problems, see Leyton-Brown et al. (2005) and Bichler et al. (2010).

Since multi-item auctions are complex and require bidders to consider multiple alternative bid options, it is important that the computer software used for communication between the bidder and the auctioneer be easy to use and understand. Good graphical user interfaces help bidders to feel comfortable that they understand the current state of the auction (they have been able to find the current price information, the items they are winning, the amount of bidding necessary to remain eligible, their dollar exposure based on what they have bid, etc.). The system must also provide easy ways for bidders to input their next moves and confirm that they have provided the system with the correct information. As the use of auctions is spreading, computer interfaces for such processes continue to improve and to provide better ways of displaying information to the users through charts, graphs and pictures. There is likely to be continued improvement in this area.

These tools do not, however, help the bidder determine the optimal combination of items to bundle as a package and the optimal number of packages to supply to the system. Since bidders face the serious problem of determining which bids are most likely to win at prices that are within the their budgets, tools that assist bidders in understanding the state of the auction is important. In both supply-chain auctions and in high-stakes government auctions (such as spectrum auctions), bidder-aided tools are often developed to assist the bidder in determining the package or packages to submit in any given round. In the case of supply-chain auctions, the auctioneer often suggests packages to the suppliers that will fit well with other bidder's bids (e.g. by either adding or removing a single item from the package, or by considering a quantity discount for supplying more of an item). Such tools have been found to be very useful and also computationally tractable; see Elmaghraby et al. (2002), Dunford et al. (2003), and Boutilier et al. (2004). Day and Raghavan (2005) and Parkes (2005) provide alternative ways for bidders to express preferences that do not require that the bidder specifying particulap packages to the auctioneer. Au2

Applications of Combinatorial Auctions

There are many examples of governments' using auctions for the allocation of valuable assets. In most of these auctions, the government is allocating a good and uses auctions to determine both the price and the allocation. Since 1994, governments throughout the world have been using simultaneous multi-round auctions for the allocation of spectrum. For spectrum, a government has the goal of allocating the good to the entities that value it the most with the hope that the bid cost will encourage the build-out of the services. To assure that there is sufficient competition in the telecommunications industry, the U.S. government has, in the past, set spectrum caps for each region. These auctions have been copied globally and are now the standard way that spectrum is allocated. Recently, a number of different package-bidding designs are being tried including the hierarchical ascending auction, the combinatorial clock auction, or the clock-proxy design. As of 2005, these auctions

Galley Proof

Comp. by: MANIKANDAN R Stage: Galleys Chapter No.: 1139 Title Name: EORMS
Date:11/8/12 Time:06:48:46 Page Number: 9

have resulted in revenues in excess of $200 billion dollars worldwide (Cramton 2005).

Within the power industry, there has also been an evolutionary movement toward auctions for the determination of who can supply power to the electricity grid and at what price. Most of the allocation is determined one day ahead of the demand. The auction reflects the unique characteristics (both physical and structural) of the industry. The allocation is determined by a complicated optimization that evaluates the demands at various nodes of the networks and prices power generation at each such node. The spot market corrects this allocation for any last minute changes due to weather, plant outages, etc. Long term contracts make this process work.

Similarly, auctions have been used to bring market-based forces to control air pollution. Here, a government entity (either nationally run or regionally administered) establishes a fixed number of tradable allowances each of which represents the legal right for its owners to emit a fixed quantity of pollution. A firm holding an allowance can emit the fixed quantity and surrender the allowance to the government, or if the firm can abate its emissions, it can profit by selling the allowance to another polluter than cannot so inexpensively abate emissions. The establishment of the fixed quantity is the cap. The exchange of allowances (credits) between polluters is the trade. See Ellerman et al. (2003) and Tietenberg (2006) for a general overview of cap and trade ideas.

The use of combinatorial auctions for the procurement of goods in services has also been growing. Some of these auctions are sealed-bid auctions, while most are moving toward multi-round auction designs. In such auctions, the providers of the goods and services are pre-screened and are then allowed to provide bids for collections of good and/or services as all or nothing packages. For a general survey of supply-chain auctions, see Bichler et al. (2005). The three applications described next highlight a few examples to show how such auctions differ from government auctions.

1. The first use of a combinatorial auction within the transportation industry was an auction conducted by Sears. Here, suppliers of freight delivery were allowed to bundle multiple lanes together into a single bid thereby allowing carriers to coordinate multiple businesses and reduce empty or low value backhaul movements. It also provided a means to incorporate surge demand contingencies into the longer (3-year) contracts, thereby lessening the need to renegotiate contracts whenever demands changed; Ledyard et al. (2002).
2. Mars Incorporated used a combinatorial auction mechanism to procure the necessary goods from multiple suppliers allowing bidders to specify complex bid structures that indicated quantity discounts, minimum supply, and multiple goods collected within a single bid. No bidder was allowed to supply more than a certain percentage of the overall quantity needed and newer suppliers were limited more severely than their suppliers they had used over a number of years. The algorithm also assured that there were multiple suppliers in the solution for each critical entity. These auctions are not simple, but work to match the needs of the procurer, Mars, with the capabilities of the suppliers (often farmers). The allocation considers geographic, volume and quality factors. The suppliers liked the auction mechanism because of its transparency, shorter negotiation time and fairness; Honer et al. (2003).
3. Motorola Corporation used auctions for the procurement of the multitude of parts needed for cellular devices. Motorola needed to reduce both the time and the effort required to prepare for and conduct negotiations with its suppliers, simplify their coordination, and optimize contract awards across sectors, in order to save costs; Metty et al. (2005).

Governments are moving toward procuring their goods and services in a similar fashion. One such example is the use of auctions to determine the suppliers of lunches in a large school system. Chile spends around US$180 million a year to feed 1,300,000 students from low income families. To improve the quality of the goods and services being provided to the school system and to save money, the government chose to assign catering contracts in a single-round sealed-bid combinational auction. This auction resulted in a transparent and objective allocation approach, thereby generating competition among firms. It also allowed the companies to build flexible territorial bids to include their scale of economies, leading to more efficient resource allocation. This new methodology improved the

Galley Proof

Comp. by: MANIKANDAN R Stage: Galleys Chapter No.: 1139 Title Name: EORMS
Date:11/8/12 Time:06:48:46 Page Number: 10

price-quality ratio of the meals with yearly savings of around US$40 million, equivalent to the cost of feeding 300,000 children during one year; Epstein et al. (2002).

In supply-chain auctions, rules are designed to assure a certain diversification in suppliers and to assure the reliability of the supply chain. In each case, are goals other than revenue maximization or efficiency that drove the auction design. In addition, the auction design must consider the nature of the investment. For spectrum, where there was both uncertainty in the long-term use of the technologies and where the cost of build-out are high, long-term leases were chosen. For energy, auctions are used for a much shorter decision problem. The U. S. Treasury uses multiple auctions for short, medium and long-term debt allocation. Oil and gas exploration must have a relatively long-term horizon where payments for wildcatting are based on the bid price and a yearly rent, whereas payments for extraction are based on bid price and royalties.

Thus, one must consider carefully the application when designing the allocation mechanism and the payment scheme. Auction theory and its use is growing because of its proven value. It provides price discovery and signals where more capacity is needed. It is often a fairer and more transparent process for the allocation of goods and services.

Conclusions

Combinatorial auctions are appropriate for problems where the bidders need to procure a collection of items that contribute to their having a viable business plan. When evaluating alternative designs, one is likely to want to satisfy the following goals:

1. The property rights are well-defined
2. Bidders are able to, through their bids, announce the entire collection of objects that they need for a given business plan
3. The auction results in maximum revenue to the seller
4. The auction results in an efficient outcome i.e. all items are collectively allocated to the bidders that value these items the most
5. The auction is perceived as fair to all bidders
6. The auction ends in a reasonable amount of time
7. The auction has limited transaction costs, i.e. the rules are not so difficult or the bidding so complicated that a straightforward bidder finds it difficult to participate
8. The auction cannot be gamed, i.e. truthful bidding is an optimal strategy for all bidders
9. The auction allows price discovery
10. The auction is computationally feasible and scalable

It is not possible to have all such attributes obtain simultaneously. For each applications, some of these goals will be more important than others. One should, however, keep all of these goals in mind when evaluating a mechanism.

In addition, the auction mechanism should consider any application-specific issues that might arise. For example, in government auctions one might want to consider how market power impacts the outcome, whether there will be sufficient participation, and whether the outcome will limit future competition in the industry. In certain situations, there may need to be a transition period that allows the market to adjust to a change in the way rights are allocated; One may have to consider the associated rights that a bidder would need to be able to use the right being sold or leased in the auction; The seller needs to determine if the rights are paid for over time or at the end of the auction; The money obtained may need to be designated for a specific use in order for the government to obtain the approval of all constituents. The auction design may also need to satisfy other social goals specific to the application (e.g. reducing emissions, increasing competition, incentivizing innovation, improving multi-modal transportation). Similarly, in supply chain auctions, a variety of goals need to be considered– quality of the goods, price, historical dependability of the supplier, among others.

See

► Auction and Bidding Models
► Integer and Combinatorial Programming

References

Ausubel, L. M., Cramton, P., & Milgrom, P. (2005). The clock proxy auction. In P. Cramton, Y. Shoham, & R. Steinberg (Eds.), *Combinatorial auctions* (pp. 113–136). MIT Press.

Galley Proof

Comp. by: MANIKANDAN R Stage: Galleys Chapter No.: 1139 Title Name: EORMS
Date:11/8/12 Time:06:48:47 Page Number: 11

Ausubel, L. M., & Milgrom, P. (2002). Ascending auctions with package bidding. *Frontiers of Theoretical Economics, 1*, 1–42.

Ausubel, L. M., & Milgrom, P. (2006). The lovely but lonely Vickrey auction. In P. Cramton, Y. Shoham, & R. Steinberg (Eds.), *Combinatorial auctions*. Cambridge, MA: MIT Press.

Bichler, M. (2011). Auctions: Complexity and algorithms. In *Wiley encyclopedia of operations research and management science*. John Wiley and Sons.

Bichler, M., Shabalin, P., & Wolf, J. *Efficiency, auction revenue, and bidding behavior in the combinatorial clock auction*. Technical Report available from M. Bichler.

Bichler, M., Davenport, A., Hohner, G., & Kalagnanam, J. (2006). Industrial procurement auctions. In P. Crampton, Y. Shoam, & R. Steinberg (Eds.), *Combinatorial auctions* (pp. 593–612). MIT Press.

Bichler, M., Shabalin, S., & Pikovsky, A. (2009). A computational analysis of linear price iterative combinatorial auction formats. *Information Systems Research, 20*(1), 33–59.

Bikhchandani, S., DeVries, S., Schummer, J., & Vohra, R. (2002). Linear programming and Vickrey auctions. In B. Dietrich & R. Vohra (Eds.), *Mathematics of the internet: E-auctions and markets* (pp. 75–115).

Bikhchandani, S., & Ostroy, J. M. (2002). The package assignment model. *Journal of Economic Theory, 107*, 337–406.

Boutilier, C., & Hoos, H. H. (2001). Bidding languages for combinatorial auctions. *Seventh International Joint Conference on Artificial Intelligence (IICAI-01)*, 1211–1217.

Boutilier, C., Sandholm, T., & Shields, R. (2004). Eliciting bid taker non-price preferences in "Combinatorial Auctions". In V. Khu-Smith & C. J. Mitchell (Eds.), *Proceedings of the national conference on artificial intelligence* (pp. 204–211). San Jose, CA.

Brunner, C., Goeree, J. K., Holt, C. H., & Ledyard, J. O. (2011). An experimental test of flexible combinatorial spectrum auction formats. *American Economic Journal: Microeconomics, 2*, 39–57.

Cason, T. N. (1993). Seller incentive properties of EPA's emission trading auction. *Journal of Environmental Economics and Management, 25*, 177–195.

Clarke, E. (1971). Multipart pricing of public goods. *Public Choice, 8*, 19–33.

Cramton, P. (2005). Simultaneous ascending auctions. In P. Cramton, Y. Shoham, & R. Steinberg (Eds.), *Combinatorial auctions* (pp. 99–114). MIT Press.

Cramton, P., Shoham, Y., & Steinberg, R. (Eds.). (2005). *Combinatorial auctions* (pp. 99–114). MIT Press.

Day, R., & Milgrom, P. (2008). Core-selecting package auctions. *International Journal of Game Theory, 36*(3), 393–407. Springer.

Day, R., & Raghavan, S. (2005). *Assignment preferences and combinatorial auctions*. Working paper, Operations and information management school of business, University of Connecticut. http://users.business.uconn.edu/bday/index.htm

DeMartini, C., Kwasnica, A. M., Ledyard, J. O., & Porter, D. (1999). *A new and improved design for multi-object iterative auctions*, Social Working Paper. Pasadena, CA: Division of the Humanities and Social Sciences, California Institute of Technology.

DeVries, S., & Vohra, R. (2003). Combinatorial auctions: A survey. *INFORMS Journal on Computing, 15*(3), 284–309.

Dunford, M., Hoffman, K., Menon, D., Sultana, R., & Wilson, T. (2003). *Price estimates in ascending combinatorial auctions*, Technical Report. Fairfax, VA: George Mason University, Systems Engineering and Operations Research Department.

Ellerman, A. D., Joskow, P. L., Montero, J., Schmalensee, R., & Bailey, E. M. (2000). *Markets for clean air: The U.S. acid rain program*. Cambridge University Press.

Epstein, R., Henriquez, L., Catalan, J., Weintraub, G., & Martinez, C. (2002). A combinatorial auction improves school meals in Chile. *Interfaces, 32*(6), 1–14.

Erdil, A., Klemperer, P., Cramton, P., Dijkstra, G., Goeree, J., Marszalec, D., Meyer, M., Milgrom, P., Pagnozzi, M., & Parkes, D. C. (2009). *A new payment rule for core-selecting package auctions*. Technical report available on Paul Klemperer's website.

Friedman, D., & Rust, J. (Eds.). (1993). *The double auction market: Institutions, theories and evidence* (Santa Fe Institute studies in the sciences of complexity, Vol. XIV). Addison Wesley.

Fujishima, Y., Leyton-Brown, K., & Shoham, Y. (1999). Taming the computational complexity of combinatorial auctions: Optimal and approximate approaches. *Proceedings of IJCAI 1999*, 548–553.

Goeree, J. K., & Holt, C. A. (2010). Hierarchical package bidding: A paper & pencil combinatorial auction. *Games and Economic Behavior, 70*(1), 146–169.

Groves, T. (1973). Incentives in teams. *Econometrica, 41*, 617–631.

Harstad, R., Pekec, A., & Rothkopf, M. H. (1998). Computationally manageable combinatorial auctions. *Management Science, 44*, 1131–1147.

Hoffman, K., Menon, D., van den Heever, S. A., & Wilson, T. Observations and near-direct implementations of the ascending proxy auction. In P. Cramton, Y. Shoham, & R. Steinberg (Eds.), *Combinatorial auctions* (pp. 415–450). MIT Press. Au3

Hoffman, K., & Menon, D. (2010). A practical combinatorial clock exchange for spectrum licenses. *Decision Analysis, 7*(1), 58–77.

Hoffman, K., Menon, D., & van Den Heever, S. A. (2008). A package bidding tool for the FCC's spectrum auctions and its effect on auction outcomes. *Telecommunications Modeling Policy and Technology: Operations Research/Computer Sciences Interfaces Series, 44*, 153–189.

Holt, C. A., Shobe, W., Burtraw, D., Palmer, K., & Goeree, J. (2007). *Auction design for selling CO_2 emission allowances under the regional greenhouse gas initiative. Regional greenhouse gas initiative*. Technical Report to RGGI.

Honer, G., Rich, J., Ng, E., Reid, G., Davenport, A., Kalagnanam, J., Lee, H. S., & An, C. (2003). Combinatorial and quantity discount procurement auctions benefit mars, incorporated and its suppliers. *Interfaces, 33*(1), 23–35.

Klemperer, P. (1999). Auction theory: A guide to the literature. *Journal of Economic Surveys, 13*(3), 227–286.

Klemperer, P. (2002). What really matters in auction design. *Journal of Economic Perspectives, 16*, 169–189.

Galley Proof

Comp. by: MANIKANDAN R Stage: Galleys Chapter No.: 1139 Title Name: EORMS
Date:11/8/12 Time:06:48:48 Page Number: 12

Klemperer, P. (2004). *Auctions: Theory and practice* (The toulouse lectures in economics). Princeton, NJ: Princeton University Press.

Koboldt, C., Maldoom, D., & Marsden, R. (2003). *The first combinatorial spectrum auction*. Ofcom Technical Report describing the results of the 2003 Nigerian Spectrum Auction, available on of com website.

Krishna, V. J. (2002). *Auction theory*. Academic Press, 200pp.

Kwasnica, A. M., Ledyard, J. O., Porter, D., & DeMartini, C. (2005). A new and improved design for multi-object iterative auctions. *Management Science, 51*, 419–4234.

Ledyard, J. O., Olson, M., Porter, D., Swanson, J. A., & Torma, D. P. (2002) The first use of a combined value auction for transportation services. *32*, 4–12.

Lehmann, D., Mueller, R., & Sandholm, T. (2005). The winner determination problem. In P. Cramton, Y. Shoham, & R. Steinberg (Eds.), *Combinatorial auctions*. Cambridge, MA: MIT Press.

Leyton-Brown, K., Nudelman, E., & Shoham, Y. (2005). Empirical hardness models. In P. Cramton, Y. Shoham, & R. Steinberg (Eds.), *Combinatorial auctions* (pp. 479–503). MIT Press.

McMillan, J. (2002). *Reinventing the bazaar: A natural history of markets*. Norton Press, 278pp.

Metty, T., Harlan, R., Samelson, Q., Moore, T., Morris, T., & Sorenson, R. (2005). Reinventing the supplier negotiation process at motorola. *Interfaces, 35*(1), 7–23.

Milgrom, P. (2004). *Putting auction theory to work*. Cambridge Press, 368pp.

Milgrom, P. (2007). Package auctions and exchanges. *Econometrica, 75*(4), 935–965.

Nisan, N. (2000). Bidding and allocation in combinatorial auctions. *Proceedings of the 2nd ACM Conference on Electronic Commerce*, 1–12.

O'Neill, R. P., Helman, U., Hobbs, B., Stewart, W. R., & Rothkopf, M. (2007). The joint energy and transmission rights auction: A general framework for RTO market designs. *Power Engineering Review, IEEE, 22*(10), 59–68.

Parkes, D. C. (2005). Auction design with costly preference elicitation. *Annals of Mathematics and AI, 44*, 269–302.

Parkes, D. C., Kalagnanam, J., & Eso, M., (2001). Achieving budget-balance with Vickrey-based payment schemes in combinatorial exchanges. *Proceedings of the 17th International Joint Conference on Artificial Intelligence (IJCAI-01)*, 1161–1168.

Parkes, D. C., & Ungar, L. H. (2000). Iterative combinatorial auctions: Theory and practice. *Proceedings of the 17th National Conference on Artificial Intelligence* (AAAI-00), 74–81.

Pekec, A., & Rothkopf, M. H. (2006). Non-computational approaches to mitigating computational problems in combinatorial auctions. In P. Cramton, Y. Shoham, & R. Steinberg (Eds.), *Combinatorial auctions*. M.I.T. Press.

Porter, D., Rassenti, S., Roopnarine, A., & Smith, V. (2003). Combinatorial auction design. *Proceedings of the National Academy of Sciences, 100*(19), 11153–11157.

Porter, D., & Smith, V. (2006). FCC license experiment design: A 12-year experiment. *Journal of Law Economics and Policy, 3*, 63–80.

Rassenti, S., Smith, V., & Bulfin, R. I. (1982). A combinatorial mechanism for airport time slot allocation. *Bell Journal of Economics, 13*, 402–417.

Rothkopf, M. H. (2007). Thirteen reasons why the Vickrey-Clarke-Groves process is not practical. *Operations Research, 55*(2), 191–197.

Steiglitz, K. (2007). *Snipers, shills and sharks: eBay and human behavior*. Princeton University Press, 298pp.

Tietenberg, T. H. (2006). *Emissions trading: Principles and practice* (2nd ed.). Washington: RFF Press.

Vickrey, W. (1961). Counter-speculation, auctions and competitive sealed tenders. *Journal of Finance, 16*, 8–37.

Wolsey, L. A. (1981). Integer programming duality: Price functions and sensitivity analysis. *Mathematical Programming, 20*(1), 173–195.

Wurman, P. R., & Wellman, M. P. (1999). *Equilibrium prices in bundle auctions*, Sante Fe Institute Working Papers (Paper: 99-09-064).

Galley Proof

Author Query Form

Encyclopedia of Operations Research and Management Science
Chapter No: 1139

Query Refs.	Details Required	Author's response
AU1	Please provide details of Klemperer (2008), Boutelier et al. (2002), Hoffman et al. (2006), Day and Raghavan (2007), Bikhchandani and Ostroy (2001), Ausubel et al. (2006), Ausubel and Milgrom (2001), Bichler et al. (2011), Bichler et al. (2010), Elmaghraby et al. (2002), Ellerman, et al. (2003), Bichler et al. (2005) in the reference list.	
AU2	Please check the spelling of "particulap."	
AU3	Please provide year of publishing for Hoffman et al. and Bichler et al.	
AU4	Please provide journal title for Ledyard et al. (2002).	