Spectrum Auctions

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Abstract Most developed countries allocate radio spectrum by auction. The Simultaneous Ascending Auction (SAA) has proven to work well for this application. Recently, new designs that allow package bidding have been proposed. These designs have only been tried in the past few years. We first provide some historical background regarding the allocation of spectrum, describe the use of the SAA design and its modifications over the past 15 years, and then highlight the new advances in combinatorial auction designs and their use for the allocation of spectrum.

1 Introduction

Many countries around the world use auctions for the allocation of radio spectrum. In order to understand the evolution of auctioning radio spectrum, one must first begin by explaining what spectrum is and how spectrum had been allocated prior to its being auctioned. Radio spectrum is the part of the electromagnetic spectrum used to transmit voice, video and data. It uses frequencies from 3 kHz to 300 GHz. Thus, it includes spectrum use for radio and TV broadcasts, paging, wireless and satellite communications, and many other important telecommunication applications. In general, most countries allocate the rights to use these airwaves as a license to provide a specific service (e.g. radio broadcast), using a specific portion of the spectrum band (e.g. the use of the 15MHz of spectrum between 950MHz and 965MHz in a specified region (e.g. a state, province or nationwide). There can be a number of restrictions on its use including the strength of the broadcast signal, the assurance that it does not create interference with adjoining spectrum regions, and other public service goals such as requiring that the provider be able to reach a certain percent-

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age of the population or a certain physical area. Spectrum is considered a national resource and is therefore allocated with concepts of social welfare in mind.

This chapter begins by first outlining some auction terms that will be used throughout the chapter. It then describes some alternative methods for spectrum allocation other than auctions and outlines the reasons for the conversion from administrative procedures to auctions. We describe in detail the most commonly used auction design - the simultaneous ascending auction (SAA) design and show how specific rules have been altered to improve its performance. Next we detail some of the newer designs that allow bidders to package licenses together and indicate a value for the “all or nothing” package. These designs remove the risk that bidders might win only part of what they need and pay too much for the resulting sub-package. We detail where these designs have been used and highlight their successes and shortcomings.

2 Some Auction Terms and Mechanisms

We begin with a very short discussion of auction design alternatives. Here we will present the terms for one-sided auctions since there have been no standardized spectrum exchanges (i.e. formal mechanisms for the *trading* of spectrum among license holders). We will also restrict our attention to the case where there is a single seller and multiple buyers, since that is the case where regulators are seeking to allocate licenses for spectrum for use by telecommunications companies. We will also assume that there are multiple items being sold and that, for at least some of the buyers, a collection of items are needed in order to have a viable business plan — they need to either aggregate licenses in order to either acquire sufficient bandwidth to offer specific services or acquire a sufficient geographic footprint. Acquiring spectrum licenses in multiple regions avoids roaming charges and allows economies of scale in terms of the build-out and marketing costs. Increases in bandwidth can increase the types of offerings carriers can provide. Thus, this chapter will highlight both multiple-item and *package-bidding auction* mechanisms that allow buyers to procure *collections* of the items that are being sold. These designs should be sufficiently general to allow bidders to express a value on a package where the collection of items may have a value greater than the individual items (i.e. the goods are *complements*), as well as on a package where a buyer can express a quantity discount for buying more of the good (i.e. the goods are substitutes).

Why does a government sell or lease valuable resources via an auction mechanism? Often the government moves to a market-clearing framework after many failed attempts at administrative negotiation with the entities that desire the use of the resources. The results of these negotiations are fraught with administrative costs and the likelihood that the items will be awarded to those that have the greatest political access rather than those that value the items the most. In many such instances, the government has no mechanism to assess the true value of the assets being leased and the public or its representatives perceive that the government is not receiving full value. Auctions allow greater participation, allow the market to set the price, and are generally perceived as fair. Often, spectrum auctions were only considered when other attempts at allocation (administrative evaluation and lotteries) failed. We will say more about this in the section on the history of spectrum auctions.

In an auction with a single seller and multiple buyers, the potential buyers wish to determine the minimum price that they must pay given that they must compete with others for the ownership of this good. From the seller's perspective, submitting goods to an auction may increase the number of buyers, thereby increasing the potential for competitive bidding and higher selling prices. Thus, an auction is a mechanism that allows the market to set the price. Since all buyers must adhere to the same set of rules, auctions are perceived as both less haphazard and fairer than if the price were set by bilateral negotiations. Most importantly, if the rules are well designed, the result will have the goods given to the entity that values them the most. When the design also encourages significant participation, the revenue provided to the seller is likely to be reasonably high.

We begin by first classifying auctions into a number of major types. One of the simplest mechanisms is the *first-price sealed-bid auction*. In this design, all bidders submit their bids by a specified date. All bids are examined and compared and the highest bid is awarded the object (or collection of objects) and pays the amount that was bid. A problem with the first-price sealed-bid auction is that the winning bidder may suffer a "*winner's curse*", i.e. the bidder may pay more than was necessary to win since the second highest bid price was far less than the winning bid amount. For this reason, first-price sealed-bid auctions encourage bidders to shave some amount from the highest amount that they are willing to pay in order not to pay more than is necessary.

An alternative is the *second-price sealed bid auction* whereby the bidder that has submitted the highest bid is awarded the object, but he pays only slightly more (or the same amount) as that bid by the second-highest bidder. In second price auctions with statistically independent private valuations[1], each bidder has a dominant strategy to bid exactly his valuation. The second price auction is often called a *Vickrey auction* [49].

Often the value of the good is not completely known. Instead, there is a *common component* to the bid value — that is, the value of the item is not independent of the others bidders. Instead, the value that others place on the asset is relevant. In such situations, each agent has partial information about the value. Many high-stakes auctions, such as antique, art and horse auctions, as well as government auctions for oil exploration, land use, and spectrum fall into this class.

When the bidders need to obtain information about the common component of the item's value, then ascending bid auctions, often known as *English* (or *open outcry*) *auctions* are used. Here, the auctioneer announces a relatively low starting price and bidders respond. A standing bid wins the item unless another higher bid is submitted. All bidders observe the previous bids and decide whether to submit a higher new bid. When someone responds with a higher bid, the auctioneer increases the

[1] In a private value auction, each bidder knows his value of the good and this information is both private and independent of the value of other bidders.

price. The auction ends when no one is willing to top the standing high bid. An alternative auction design is the *Dutch auction* whereby the price is set very high and the price is gradually lowered by a clock until it is stopped by some bidder that is willing to buy some or all of the (identical) items up for sale at the current price. The quantity bought by this bidder is deducted from the total amount available and the auction resumes with the auctioneer continuing to reduce the price until another bidder indicates a wiliness to buy some quantity of the item at the announced price. The name derives from the fact that tulips were sold by the Dutch via this mechanism. This auction mechanism is still employed for many agricultural products where many identical (fungible) items are being sold and the bidders determine both the price and the lot size.

Finally, for a large group of identical objects, a variation of the ascending auction is one in which the auction price ascends until there are no new bids. At the end of the auction, all buyers pay the lowest amount bid by any of the winning bidders. Such auctions are called *all-pay one-price auctions*, or *uniform price* auctions.

Another important auction format issue is whether the auctioneer sells each of the items sequentially or auctions all items simultaneously. With sequential auctions, bidders must guess the resulting prices of future auction items when determining how to respond to prices in the current auction. If a bidder needs multiple items and the items are auctioned sequentially, then the bidder is faced with the issue of risking winning only part of what is needed. One mechanism that has been used to overcome this problem is to first auction related items individually (e.g. auction each chair and the table of a dining room set separately) and then ask if there is someone who is willing to pay more for the set than the sum of the individual items. But, this approach assumes that there is a unique package that may have a greater value than the individual components. Alternatively, all items can be auctioned *simultaneously*. We present one such auction design: *the simultaneous ascending auction* (SAA) in detail because that design has been the most popular in auctioning spectrum.

We now specify a few additional rules of an auction mechanism that can have significant impact on its success. Firstly, an ascending auction can have discrete rounds or bidders may be allowed to provide bids continuously. With the continuous approach, there must be a stopping rule that determines when the auction ends. Auctions, such as eBay are continuous auctions and specify a fixed time when the auction will end. Fixed stopping times encourage bidders to stay silent until very late in the auction and then present last minute bids in the hopes that other bidders do not have time to respond. Such bidding activity is called *sniping*. Conversely, if the auction has discrete rounds, then one may need *activity rules* that force bidders to participate throughout the auction. These activity rules can help price discovery. In an ascending multi-round auction design, bidders see the price of all items during the auction and adjust their strategies and business plans as the auction progresses.

Other questions in setting up an auction design are how to set the opening bid price (in an ascending auction) and, in either sealed-bid or open-outcry auctions, whether or not to set a reserve price. If the auction does have a reserve price - for fear that the item might otherwise be sold at too low a price - the auctioneer must also decide whether to have that reserve price revealed before the auction begins or have it disclosed only when it need be enforced.

In many high-stakes auctions, prior to the beginning of the auction, the auctioneer has a qualification stage. In this stage, bidders must prove that they are qualified both monetarily and have the skills to fulfill whatever obligations go along with the leasing/owning of the property.

As stated earlier, throughout this chapter, we will assume that we are seeking an auction format that is appropriate for problems where bidders need to procure a collection of items in order to have a viable business plan. When describing the history of spectrum auctions, we will highlight the success or failure of specific designs at satisfying some or all of the possible goals of a telecommunications regulator. Such goals may include:

1. The bidders understand what they are buying, i.e. the property rights are well defined. A fully-described property right will include the ways in which the item can be used, the time-period of use, the termination process for misuse or lack of payment, the right of the owner/lessee to sub-let/sell the right, the obligations of the lease holder, etc.
2. Bidders are able to, through their bids, announce the entire collection of objects that they need.
3. The auction results in reasonable revenue to the seller.
4. The auction results in an efficient outcome i.e. all items are collectively allocated to the bidders that value these items the most.
5. The auction is perceived as fair.
6. The auction ends in a reasonable amount of time.
7. The auction has limited transaction costs, i.e. the rules are not so difficult or the bidding so complicated that a straightforward bidder finds it difficult to participate.
8. The auction design discourages gaming and signaling.
9. The auction allows price discovery, since such auctions often bring more participation.
10. The auction is computationally feasible and scalable.
11. The auction satisfies, through its rules, any social welfare outcomes that are desired by the public entity selling or leasing the item.

It is not possible to have all such attributes simultaneously. For certain applications, some of these goals will be more important than others. For more on what really matters in auction design, we recommend the paper by Klemperer [32] who maintains:

> What really matters in auction design are the same issues that any industry regulator would recognize as key concerns: discouraging collusive, entry-deterring and predatory behavior. In short, good auction design is mostly good elementary economics.

In addition, the auction mechanism should consider any application-specific issues that might arise. For example, an auction design might consider how market power impacts the outcome, whether there will be sufficient participation, and whether the outcome will satisfy specific social welfare goals. In certain situations, there may need to be a transition period that allows the market to adjust to a change in the way rights are allocated. One may need to consider the associated rights that a bidder would need to be able to use the right being sold or leased in the auction. For expensive goods, buyers may need to pay for the lease over a period of time. The money obtained from the auction may need to be designated for a specific use in order for the government to obtain the approval of all constituents. The auction design may also need to satisfy other social goals specific to the application (e.g. increasing competition, incentivizing innovation, assuring access to most or all of the citizenry, precluding interference with other licenses and technologies, assuring that international treaties are satisfied).

The allocation of spectrum has many of the social-welfare issues described above and the design of the auction must keep these goals in mind. In the next two sections, we will first detail the New Zealand auction — the very first auction to allocate spectrum. Then, we describe the evolution of spectrum allocation in the U.S. from administrative procedures to auctions and then describe the simultaneous ascending auction in detail. This auction design is now used throughout the world and is based on concepts from mathematical game theory. In this chapter we present a summary of this evolution, for more reading on this subject, we recommend: McMillan [39] on the history of markets, Krishna [35] on auction theory, Steiglitz [48] on the success and pitfalls of eBay auctions, Klemperer [33] on auction theory and practice, and Milgrom [41] for a more detailed history and theoretical underpinnings for the simultaneous ascending auction design chosen by the US Federal Communications Commission (FCC).

3 The First Spectrum Auction: The New Zealand Experiment

The first auction of radio spectrum took place in 1990 in New Zealand. An economic consulting firm, NERA, recommended using a second-price sealed-bid auction design. This auction was for multiple nationwide television licenses, each of 8 MHz. Each license was considered essentially identical and a separate sealed-bid auction was held for each license where all bids for all licenses were submitted at the same time. Thus, participants in the auction needed to submit a separate bid simultaneously for each license. Bidders who only wanted one license were uncertain as to how to bid. Do they bid in only one auction, or in all auctions? If only one auction, which one? If many entities bid in some auctions but not all, then in the auction where there is significant competition the license is likely to cost more than some other (identical) license that had less competition. The results of the New Zealand auction illustrate precisely the problem that bidders encountered: one bidder bid on all six licenses but won only one, while another bid for just one license, won that license, and paid less than the bidder who bid on all six licenses. The license prices ranged from NZ$410,000 to NZ$100,000. Since the New Zealand government published the bid amounts of each bid, there was a public outcry when it was disclosed that a bidder was willing to pay NZ$2,371,000 for a license but only paid NZ$401,000 because of the second-price design.

McMillan [38] chronicles two other New Zealand auctions where bidders paid far less than their bid price. In one auction, a bidder placed a bid for NZ$100,000 but paid only NZ$6 (the second-highest bid) and in another, a bidder placed a bid of NZ$7Million and paid only NZ$5,000. In 1991 and continuing through 1994, the New Zealand government reverted to first-price sealed bid auctions preferring winners to be exposed to the winner's curse rather than to have the government receive virtually nothing for licenses.

As Milgrom [41] points out, there were a number of auction design alternatives that the New Zealand government could have chosen instead of the design chosen. For example, within the sealed-bid approach, each bidder could have been required to submit a demand schedule indicating, for each alternative amount of bandwidth, the amount it was willing to pay. In this design, all licenses would have been considered simultaneously within a single auction. Klemperer argues that an alternative design, namely the Dutch auction could have been used, whereby the auction starts at a high price and the price is reduced until a bidder indicates that it is willing to buy licenses at that price. That bidder then gets to announce the quantity it wants and the auction continues with the remaining licenses. These suggested alternatives highlight that there are many auction choices and one must consider the pros and cons of each. In 1994, the U.S. Federal Communication Commission (FCC) chose a truly new design for its first spectrum auction.

4 Evolution of the US Simultaneous Ascending Auction (SAA) Design

Prior to the allocation of spectrum by auction, the FCC allocated spectrum through comparative hearings whereby regulators would review and compare applications provided by a variety of private enterprises in order to determine which of the applicants would put the spectrum to its best public use. This administrative process was extraordinarily lengthy. The process required that the petitioner hire a large staff of lawyers, lobbyists, engineers and accountants to justify its claims that it is the most qualified to provide the services. The regulators, as well, needed large staffs to be able to first create the comparative criteria for applicants to use when developing their applications and then to evaluate the claims made. There were often lengthy appeals processes and court battles over these licenses. This administrative process is often referred to, in the economic literature, as a "beauty contest" because of its lack of transparency and because the decisions were based on the often not fully understood preferences of a small group of people.

By 1982, the FCC and Congress wanted to encourage the use of a new technology: cellular phones. Congress worried that the administrative process would delay the introduction of these technologies and therefore passed legislation that required that cellular licenses be allocated by lottery. Specifically, all applicants who could provide documentation that they had the funding to build out a cellular business would be placed into a pool. The government would choose among this applicant pool randomly.

This lottery process allocated the licenses very quickly - the essential goal of the legislation. However, the potential for large gains from winning a license resulted in over 400,000 applications for cellular licenses and many of the winners were speculators with no experience in telecommunications. A secondary market ensued for the active re-selling of licenses. But, for those who wanted to create a nation-wide cellular license, it was difficult to accumulate the many small regional licenses through bilateral negotiations with multiple sellers. In addition, a flurry of news reports highlighted specific people who made substantial profits by applying for a license but with no experience or interest in being part of the telecommunications industry[2].

To correct this revenue loss to the US government and to assure that the licenses were awarded quickly to those that would build-out the new technologies, Congress, in 1993, authorized the licenses of spectrum by auction. It specified that the first auction must take place by July 1994. The FCC was instructed to create procedures that:

A. Encourage the development and rapid deployment of new technologies, products and services for the benefit of the public, including those residing in rural areas, without administrative or judicial delays;
B. Promote economic opportunity and competition and ensures that new and innovative technologies are readily accessible to the American people by avoiding excessive concentration of licenses and by disseminating licenses among a wide and variety of applicants, including small businesses, rural telephone companies and businesses owned by members of minority groups and women;
C. Recover for the public a portion of the value of the public spectrum resource made available for commercial use and avoidance of unjust enrichment;
D. Promote efficient and intensive use of the electromagnetic spectrum; and
E. Ensure that, in the scheduling of any competitive bidding process, an adequate period of time is allowed, before issuance of bidding rules, to permit notice and comment on proposed auction procedures, and, after issuance of bidding rules, to ensure that interested parties have a sufficient time to develop business plans, assess market conditions and evaluate the availability of equipment for the relevant services[3].

With these goals in mind, the FCC began obtaining advice from game-theoretic economists about auction design. John McMillan was hired as a consultant to the FCC while a number of economists provided written filings to the FCC. Evan Kwerel, an economist within the FCC, working with the FCC Chairman Reed Hundt,

[2] One such group consisted of dentists in Massachusetts who won a license and immediately sold it for $41 million. For more on the history of the FCC auctions and their predecessor administrative and lottery approaches, see Milgrom [41] and McMillan [39].

[3] Section 309(j) of the Communications Act, 47 U.S.C.

focused on getting it right the first time, since as he stated in the preface to Milgrom [41]:

> It was my view that whatever method was used in the first FCC auction, if it appeared successful, would become the default method for all future auctions.

The FCC hoped to encourage participation by new entrants while simultaneously allowing existing cellular licensees to acquire holdings that were complementary to their existing holdings. Since different business plans required very different aggregations, it was not clear how to optimally subdivide the country and the sequential auctioning of different lots would hamper a bidder's ability to aggregate licenses of complementary regions.

The FCC chose a novel auction design by choosing to auction all licenses simultaneously. Bidders could bid on any item until the bidding stopped on all items. Within each round, the auctioneer examines all bids on a given item, and announces that a bidder is "provisionally winning" if that bidder has the highest bid on that item in that round. A "provisionally winning bidder" is responsible for paying his bid price unless outbid by another bidder in a later bidding round. When there are two bidders with the same high bid, the provisionally winning bidder is chosen randomly among those with the high bid.

Thus, bidders examined the current round prices of all of the licenses and choose a set of licenses that fit their business plan. Prices of substitutable licenses would remain relatively similar, and a bidder had the flexibility to alter their business plans as licenses rose in value. Although this auction design did not allow bidders to package licenses into "all or nothing" bids, bidders had the flexibility to move among different band plans and collect a sufficient group of licenses to develop usable business plans.

Another novel feature of this design was the concept of an *activity rule* whereby each bidder is required to participate throughout the auction. The activity rule was based on a "bidding unit" concept whereby a bidding unit relates to both the population and the spectral size (as measured in megahertz) of the band (often referred to as "MHz pops"). Thus, licenses were ranked by size, a license covering New York, for example, had more bidding units associated with it than a license covering a rural area in Montana. A bidder had to secure bidding units prior to the start of the auction by way of a refundable upfront payment. Based on this upfront payment, bidders were "eligible" to bid on no more than a given number of bidding units. The bidder was then forced to consider all of its bids and assure that the number of bidding units associated with the bids placed in a round met an activity requirement (normally stated as a required percentage of a bidder's eligibility). If the bid did not meet this requirement, then the bidder was threatened with a reduction in its eligibility, thereby limiting the number of bidding units that it could bid on in the future. By forcing bidders to bid each round or lose eligibility, the relative value of items (i.e. price information about the items for lease) was provided to bidders each round. There were also penalties imposed for bidders that defaulted on a winning bid. Thus, as the auction progresses and prices rise, the activity rule forces bidders to place binding pledges at the current prices or leave the auction. This rule precludes bid sniping (staying silent until very late in the auction and then presenting last minute bids in the hopes that other bidders do not have time to respond).

Until recently, the FCC auctions were completely transparent. That is, bidders and other interested parties could see the identities and bids of all bidders in each round. Thus, all participants could verify that the rules were followed. If problems existed, they could be found and resolved. It was argued that transparency was essential, both because of the newness of the auction design and because bidders needed to establish values based on a number of technological issues: they needed to be sure that their customers could roam onto other networks that used similar technologies and that they needed to understand the interference issues that might arise based on who owned the licenses adjacent to the considered license. However, such transparency gave bidders valuable price information at no cost and provided mechanisms for colluding and bid signaling. Recently, the auction rules were changed so that only the provisionally winning bid amounts are disclosed. Bidder identities are no longer revealed until the end of the auction. For more on the gaming that is possible when bidder identities are provided see Cramton and Schwartz [16].

The legislation required that the FCC encourage participation by small businesses, minorities and new entrants. Instead of creating set-asides for such groups, the auction design chose to provide such groups with bidding credits. The idea was that small bidders might find it difficult to acquire the capital to compete with major telecommunications companies. To overcome this handicap, small companies and minority and women-owned businesses (referred to as "designated entities") would be given a "credit". If a designated bidder won a license at a given price they would only pay some fraction (e.g. 90%) of the price bid. In this way, both small and large bidders would be bidding within the same auction, thereby encouraging participation and competition. In the FCC auctions, the bidding credits have ranged as high as 40% and as low as 10%.

The FCC was also concerned with limiting the amount that a single participant could acquire in the auction. There was fear that "deep pocket" bidders could become a monopoly (or duopoly) player in important markets. To assure that this was not possible, a spectrum cap was placed in each geographic area. Spectrum caps assured that incumbents were significantly inhibited from bidding in regions where they already had large holdings and insured that, in some major markets, the auctions would end with new entrants.

The Simultaneous Ascending Auction (SAA) design has proven extraordinarily successful and been adopted worldwide, resulting in revenues in excess of $200 billion dollars (Cramton [11]). The rules were originally proposed by Paul Milgrom, Robert Wilson and Preston McAfee and have been refined over time when problems arose with the original design. We highlight some of the problems and the refinements to the rules.

At first, the FCC allowed bidders to bid any whole dollar amount above a minimum acceptable bid amount on any license during a given round. This bid flexibility was intended to allow bidders to increase the speed of the auction by pushing prices up quickly when it was known that the opening price of a given license was considerably lower than its eventual selling price. However, this flexibility also allowed bidders to signal to other bidders and to punish bidders if they did not agree to collude. Specifically, bidders were able to engage in signaling by attaching identifiers in the trailing digits of their bids (called "code bids") to signal another bidder that it would be punished if it continued to bid on a certain license. A bidder could place a very large bid price on a license that it did not want, but knew another bidder wanted, and then subsequently withdraw the bid, thereby signaling that it was willing to stay off the rival's properties if the other bidder reciprocated by staying off the signaler's most desired licenses. In order to stop this behavior, the FCC implemented "click-box bidding" whereby the bidder could only click on the price set by the FCC. In order to move the auction along, the FCC allowed bidders to place a limited number of "jump bids", i.e. the bidders could choose one of nine possible increment bids — the minimum increment or any of eight other increments, each 10% above the minimum increment. In more recent auctions, the FCC has limited jump bidding to fewer choices, still concerned that the jump bids could be used as signals.

The original design allowed withdrawal of bids in order to allow a bidder who only won some of what they wanted to withdraw from this partial aggregation. However, as already discussed, bidders could and did bid on a rival's license in one round and withdraw it in the next as if to say, "I will stop bidding on your license if you stop bidding on mine." To preclude this signaling, the FCC has severely limited the number of withdrawals that are allowed throughout the auction. The changes in these rules (click-box bidding, and limited withdrawals) have limited the amount of signaling, but they did not completely eliminate the problem. In a further attempt to deter such signaling, the FCC has recently removed the reporting of bidder identities each round making it nearly impossible to single out and punish any given bidder.

Other enhancements that have taken place since the inception of the SAA auction design include: (a) eliminating installment payments which encouraged speculative bidding, (b) using license-specific bid increments whereby items with significant bidding activity have larger bid increments thereby speeding the auction along, (c) increasing the number of rounds per day, and (d) setting the minimum opening bid amounts higher. For more on the history and issues with the SAA, see Milgrom [41], Cramton [12], Porter and Smith [46], and Wilkie [50].

One major shortcoming of the above design is that while it does allow bidders to bid on multiple items at the same time, it does not allow a bidder to describe the synergy value of a package of licenses. When the winner of each item is determined independently, bidding for a synergistic combination is risky. Without the ability for a bidder to state that his value for a collection of items is greater than the value of the sum of the individual items, an *exposure problem* can exist. The bidder may fail to acquire key pieces of the desired combination, but pay prices based on the synergistic value. Alternatively, the bidder may be forced to bid beyond his valuation on some pieces in order to secure the synergies and reduce its loss from being stuck with only the less valuable pieces. Bidding on individual items exposes bidders seeking synergistic combinations to aggregation risk. To mitigate this exposure problem, the FCC allowed bidders the option of withdrawing bids a limited number of times, so they could back out of failed aggregations. Another feature that mitigated the exposure problem was that there were multiple bands of spectrum that could be substituted (bands of spectrum close to one another in the same region). An active secondary market allowed trading to correct for incomplete package formations.

Recently, the FCC has considered altering the overall auction design. In 2000, it proposed a simultaneous multi-round ascending *package-bidding* design for the 700 MHz band of spectrum (the spectrum freed due to the conversion from analog to digital television). The package-bidding design proposed for that auction was not scalable but was easily computable for an auction of 12 licenses. The spectrum for this auction was based on the conversion of analog TV channels to digital, and the auction was postponed because that conversion took longer than originally anticipated. By 2008, when the conversion was imminent, considerably more spectrum had become available and the FCC decided to include other spectrum bands within the auction. In deciding the bandplan, the FCC chose configurations for the different bands that made the movement from one band to another difficult.

Specifically, the FCC decided to divide these spectrum bands into various regional configurations: one band was a nationwide license, one band had twelve regional licenses, two bands had 176 licenses and one had 734 licenses, yielding a total of 1099 licenses[4]. For this large number of disparate license configurations, the package-bidding auction postulated in 2000 was not feasible. The number of alternative packages that a bidder could create was exponentially huge and unworkable. Instead, the FCC chose a modified package-bidding design where only the licenses on one band with twelve regions (the "C" block) could be packaged and, even on this band, the FCC pre-configured the allowable packages. In each of the other bands, no packaging was allowed.

Other rules changes for this auction included: (a) having only two possible jump increments, (b) making all bidding anonymous (except for the announcement of the provisionally winning bids), (c) forcing "open access" rules for the C band, and (d) adding multiple rules for the nationwide license that related to public-safety use.

The results of this auction were mixed. The revenue from this auction exceeded $19 billion. However, the nationwide license did not reach the reserve price and remained unsold, probably because of the onerous public-safety rules surrounding that license. AT&T and Verizon won most of the licenses being auctioned, thereby strengthening their holdings in the cellular business — the cap on spectrum holdings had been eliminated prior to this auction. The varying sizes of the blocks auctioned made it particularly difficult to move across bands and thereby hindered the substitutability property that had been prevalent in prior auctions. Because the blocks were partitioned differently with different technology rules and specifications, it was difficult for bidders to substitute seemingly identical spectrum as prices rose. The result was that bands with similar characteristics, in terms of MHz and regions, sold for significantly different amounts. Cramton [15] argues:

> In most instances, spectrum lots covering the same region in adjacent frequencies are nearly perfect substitutes. The bidder primarily cares about the quantity of spectrum in MHz it has

[4] This auction was relabeled Auction 73 whereas the original 12-license combinatorial auction was labeled Auction 31.

in the region, rather than the exact frequency location. Moreover to minimize interference problems, bidders prefer contiguous spectrum within a region. In this setting, it makes sense in the initial stage to auction generic spectrum.

Pressure from varying groups argued against having all bands have the same regional configuration and the end result was that bidders found it difficult to move from band to band during the auction.

In addition, unlike many past FCC auctions, this auction saw few new entrants winning large holdings of spectrum. Both the non-substitutability of the bands and the FCC removal of the spectrum caps contributed to the result that the two largest cellular carriers acquired significant new spectrum holdings. By comparison, the Canadian government chose, in its 2008 SAA-design 3^{rd} Generation spectrum auction, to set aside specific bands for designated entities and to also allow these designated entities to bid on blocks of spectrum that were not set aside. The Canadian auction ended with prices across all blocks being valued similarly.

We point out the differences in results between the Canadian and FCC auctions to suggest that it is not only the auction mechanism itself, but also the band plan and the rules associated with participation that can impact the success of the auction. In making such decisions, the regulator first determines how much spectrum will be available for auction, determines how substitutable the spectrum is (e.g. are there any encumbrances on some of the spectrum? Are there any problems with interference for the spectrum closest to already allocated spectrum?). Next, the regulator must determine what the allowable uses will be and then, one must decide the appropriate regional specification and bandwidth size for each license. Finally, the regulator must determine the auction mechanism and the specific rules within that auction design.

As described earlier in this chapter, the New Zealand government first tried a second-price sealed-bid auction (without reserve prices), then a first-price sealed-bid auction. Not being happy with the results of either of these two approaches, the government went to an SAA auction in 1994. Similarly in 1993, the Australian government had some adverse experience in the auctioning of licenses for satellite-television services. A first-price sealed bid auction was used to auction two satellite-television service licenses. The winners of the two licenses had bid very high prices, won the licenses and then defaulted. When the licenses were re-allocated to the next highest bidders, they too defaulted and a cascade of defaults ensued until a bidder was found that could pay for his/her bid. The final prices actually paid were on the order of 100 million dollars less than the original bids. The Australian government decided to use a simultaneous ascending-price multiple-round auction mechanism (with bid default penalty mechanisms) in 1997, and continues to use this auction format for spectrum license assignment. Due to these and other similar experiences with sealed-bid auctions, most countries today use some form of the simultaneous ascending package bidding design.

But SAA designs are not foolproof. In 2000, the German government held an auction for third-generation (3G) spectrum use using a SAA auction design. Although the auction brought $45 billion to the German government, these prices (set during the dot-com bubble) were perceived to be too high and the entities that were awarded the licenses found it difficult to obtain financing for the build-out of the networks. A similar result occurred in the UK auction for 3G use held during the same period. Thus, what looked like successes at the end of the auction may not have satisfied the government's long-term goal of quick build-out of new cellular technologies. One problem with these auctions was that they were the first auctions for 3G spectrum in Europe. If major telecommunications corporations did not get licenses in these countries, then their ability to have a large footprint throughout Europe would be impaired. Thus, the timing of auctions can have a significant effect on outcomes.

There are those that argue that one problem with ascending auctions is that bidders set their valuations within the auction based on the price signals of others. Public outcry auctions encourage such unfettered exuberance, since the auctioneer represents the seller who wishes to maximize revenue. However, when the government is selling assets for social welfare goals, overbidding can be detrimental to these goals. For more on the results of the European Auctions and their differing results, see Cramton [11] and Klemperer [33].

To conclude: a regulator must consider many issues when designing a spectrum auction. First, one must be careful to consider the consequences of the band plan. Slight changes in auction rules can lead to unintended consequences. Over the 15 years that the FCC has been involved in auctions, the rules have changed to accommodate new circumstances or to reduce any observed bidder actions that were believed to be harmful to the outcomes.

Today, spectrum auctions take place throughout the world. Sometimes the decision to move from administrative processes to auctions occurs when a scandal associated with the administrative process forces the country to create a more transparent approach to the allocation; other reasons may include that the regulator wants to assure that the public receives a portion of the value of the resource made available for commercial use; that the new technologies reach the public quickly, or that there is sufficient competition within the market.

Recently, newer auction designs are being tested both in laboratories and by regulators. We detail the two designs that have been proposed for spectrum use.

5 Combinatorial Auction Designs and their Use for Spectrum Allocation

One of the strongest arguments against the simultaneous multi-round auction is that the design has an exposure problem whereby a bidder only wins part of what he needs. In a simultaneous ascending-round auction, the bidder may be stuck with items that are valued at less than the price won because the other complementary goods were not won. To overcome the exposure problem, many have proposed auction designs that allow bidders to construct packages of subsets of the items. Such auctions are called combinatorial auctions. In this chapter, we will only discuss auction designs that have been used to allocate spectrum. For the interested reader, we suggest the following papers that describe the use of combinatorial auctions in other applications: bus services for the London Transportation System (Cantillon and Pesendorpher [9], supplying meals to school children in Chile (Epstein et al. [23]), supply-chain procurement auctions (for a Mars Candy Company Auction see Hohner et al. [30]; for Motorola's Procurement Process see Metty et al. [40]; for Sears' Combinatorial Auction, see Ledyard et al. [37]). For a survey of combinatorial auctions used in the power industry, see Kaglannas et al. [31]. For a detailed discussion of alternative package-bidding auction designs and their applications see the text *Combinatorial Auctions* edited by Cramton et al. [17].

5.1 Sealed-bid Combinatorial Designs

A relatively simple idea is to allow package bids to be submitted within a sealed-bid framework. In this setting, one must determine the set of bidders that maximizes the revenue to the seller. Since package bids often overlap, determining the collection of package bids that do not overlap and maximize revenue is a combinatorial optimization problem known as the *winner determination problem*. Thus, each bidder provides a collection of bids to the auctioneer and the auctioneer selects among these bids the collection of bids that maximizes revenue. The simplest of the formulations for the winner determination problem can be specified mathematically as:

$$[\text{WDP}] \quad \max \sum_{b=1}^{\#Bids} BidAmount_b x_b$$

subject to

$$Ax = 1 \tag{1}$$

$$x \in \{0,1\} \tag{2}$$

where x_b is a zero-one variable indicating if bid b is winning or not; and A is an m x n matrix with m rows, one for each item being auctioned. Column A_j describes package bid j, where $A_{ij} = 1$ if package j contains item i, and zero otherwise. Constraint set (1) specifies that each item can be assigned exactly once. Each of the n columns represents a bid where there is a one in a given row if the item is included in the bid and zero otherwise. Constraint set (1) are equations rather than $\leq$ constraints since the regulator provides m bids (one for each item) at a price slightly below the minimum opening bid price. In this way, the regulator will keep the item rather than allow it to be won by a bidder at less than the opening bid price.

One issue in a combinatorial auction design is the bidding language used. A simple idea is that the bidder submits a collection of package bids and the bidder can win any combination of these bids as long as each item is awarded only once, referred to as the OR language[5]. The problem with this language is that it creates a new type of exposure problem - that of winning more than the bidder can afford. When multiple bids of a single bidder can be winning, it is incumbent on the software to highlight the maximum exposure to the bidder. This calculation requires that a combinatorial optimization problem be solved for each bidder that calculates the dollar exposure, creating new computational issues for the auctioneer.

Fujishima et al. [25] propose a generalization of this language called OR* that overcomes this problem. Namely, each bidder is supplied "dummy" items (these items have no intrinsic value to any of the participants). When a bidder places the same dummy item into multiple packages, it is telling the auctioneer that he wishes to win at most one of these collections of packages. This language is fully expressive as long as bidders are supplied sufficient dummy items. We note that this language is relatively simple for bidders to understand and use, as was shown in a Sears Corporation supply-chain transportation auction. In that auction, all bids were treated as "OR" bids by the system. Some bidders cleverly chose a relatively cheap item to place in multiple bids thereby making these bids mutually exclusive, see Ledyard et al. [37]). There have been a number of alternative bidding languages that have been proposed. We refer the reader to the research of Boutilier et al. [6], Fujishima et al. [25] and Nissan [42] for complete descriptions of the alternative languages.

A simple bidding language to evaluate from a theoretical game-theory perspective, and the one that all current spectrum bidding software uses specifies that all bids of a given bidder are mutually exclusive. This language also removes the dollar exposure problem, since the maximum that a bidder can possibly pay is the highest bid amount of any of its bids. The problem with this language is that it places a new burden on the bidder: the bidder is forced to enumerate all possible combinations of packages and their associated value. Clearly, as the number of items in an auction increase, the number of possible bids goes up exponentially. When the XOR bidding language is used the winner determination problem becomes:

$$[\mathrm{WDP_{XOR}}] \ \max \sum_{b=1}^{\#Bids} BidAmount_b x_b$$

subject to

$$Ax = 1 \tag{3}$$

$$\sum_{b \in S_B} x_b \le 1 \qquad \text{for each Bidder } B, \tag{4}$$

$$x \in \{0,1\} \tag{5}$$

where S_B is the set of bids of bidder B, and constraint set (4) specifies that at most one of these bids can be in the winning set.

For a first-price sealed-bid combinatorial auction, one need only solve the winner determination problem to determine the winning bidders and the amounts that each

[5] The formulation WDP has the assumption of the OR language. Below we present the formulation when bids of a given bidder are mutually exclusive, labeled $\mathrm{WDP_{XOR}}$.

pays, since they pay that which they bid. But, as with the case of a single item auction, the bidder can experience the winner's curse of having paid too much for the items won in a first-price sealed-bid combinatorial auction.

We now consider second-price sealed bid auctions when bidders can bid on all or nothing packages In this case, we again look toward the Vickrey solution and generalize the idea when there are multiple items and the bidder wants to win "all or nothing" packages. To determine the winning bidders, one solves the same winner determination problem as one does for the first-price sealed-bid case but the winners do not necessarily pay what they bid. Instead, one calculates the marginal value to the seller of having this bidder participate in the auction. To do this, for each winning bidder, one calculates the revenue that the seller would receive when that bidder participates in the auction and when that bidder does not, i.e. when none of the bids of this bidder are in the winner determination problem. The difference in the two objective function values is known as the *Vickrey-Clarke-Groves discount*[6], and the bidder pays his bid price minus the discount. When winners pay this amount, the auction is known as the Vickrey-Clarke-Groves (VCG) Mechanism.

Although it can be shown that the VCG mechanism encourages truthful bidding, it is almost never used in practice. For a complete list of reasons for it being impractical, see Rothkopf [47] and Ausubel and Milgrom [3]. In essence, the prices provided by this mechanism may be very low. Worse yet, when items have complementary values — i.e. the package is worth more to the bidder than the sum of the values of the individual items — the outcome may price the items so low that there is a coalition of bidders that would prefer to renege on the auction and negotiate privately with the seller, and the seller may respond by reneging on the sale since both the seller and the coalition of buyers will be better off. Ausubel and Milgrom [2] argue that prices should be set high enough so that no such coalitions exist. In game theoretic terms, the prices are set such that the outcome is in the core of a coalitional game. These authors introduced an auction design known as the "ascending proxy auction" in which the bidders provide all bids as if in a sealed-bid auction. Each bidder is provided with a proxy that bids for the bidder in a straightforward manner during an ascending auction. The proxy only announces bids that maximize the bidder's profit (i.e. bid price - announced price) in any given round. The auction continues as an ascending package-bidding auction until, in some round, there are no new bids. Thus, the auction simulates through proxy bidders an ascending auction where the increment in each round is infinitesimally small and each bidder, through the use of its proxy, bids in a straight-forward manner. This auction design is very similar to the iBundle design of Parkes [43].

Hoffman et al. [29] provide a computational approach toward speeding up the calculations associated with this auction design and Day and Raghavan [20] provide an elegant mechanism to obtain minimal core prices directly. The direct mechanism of Day and Raghavan sequentially solves winner determination problems to determine losing coalitions that could supply more revenue to the seller at the current

[6] The three authors, Vickrey [49], Clarke [10] and Groves [27] wrote separate papers each producing certain attributes that this auction design has as it relates to incentivizing bidders to reveal their truthful value of the goods demanded.

prices. To begin, the winner determination problem is solved with the bids of losing bidders at their announced amounts bid (i.e. treated as first prices), and the bid prices of any winning bidder's bids are set at the amount bid minus the VCG discount for that bidder. We refer to this winner determination problem as $\text{WDP}_{\text{Core}(\tau)}$ and state it as follows:

$$[\text{WDP}_{\text{Core}(\tau)}]$$

$$\max z^{\tau} = \sum_{B \in NW} \sum_{b \in S_B} BidAmount_b x_b + \sum_{B \in W} \sum_{b \in S_B} (BidAmount_b - Discount_B^{\tau}) x_b$$

subject to

$$Ax = 1 \tag{6}$$

$$\sum_{b \in S_B} x_b \leq 1 \qquad \text{for all Bidders } B, \tag{7}$$

$$x_b \in \{0,1\} \qquad \text{for all bids of all Bidders.} \tag{8}$$

This is the first iteration of an iterative algorithm, so $\tau = 0$. S_b is the set of bids of a given bidder B, W is the set of winning bidders and NW is the set of non-winning bidders, and $Discount_B^{\tau}$ is the VCG discount for winning bidder B. If the solution to this optimization problem yields revenue, z^0, greater than the VCG mechanism would provide, then the prices of the winning bid set are raised so that the total price paid by winning bidders is equal to this new revenue. We denote C^t as the set of winning bidders in $\text{WDP}_{\text{Core}(\tau)}$. To determine the new prices to be considered in iteration $\tau + 1$, one must be sure that any winning bidder that forms part of this blocking coalition does not have its price raised from its prior price since it would not be willing to join a coalition if it were to lose revenue relative to its prior offer by the seller. We therefore have the following linear optimization problem to solve at each step t of the algorithm:

Cutting Plane Approach for Finding Prices (Stage t):

$$\min \ \Theta(\pi)^t = \sum_{j \in W} \pi_j^t$$

subject to

$$\sum_{j \in W \setminus C^{\tau}} \pi_j^{\tau} \geq z(\pi^{\tau}) - \sum_{j \in W \cap C^{\tau}} \pi^{\tau} \qquad \tau \leq t, \tag{9}$$

$$\pi_j^{VCG} \leq \pi_j^{\tau} \leq \pi_j^{bid} \qquad j \in W, \tag{10}$$

where π_j^t will be the prices that will eliminate the coalition found in $\text{WDP}_{\text{Core}(\tau)}$ for $\tau = t$. Each of the constraints in (9) is a "cut" that forces the prices higher at each iteration t. The constraint assures that the total price paid is greater than or equal to the revenue $z(\pi^t)$ for each $t \in T$ and that the price paid by any winning bidder that is part of the coalition found at iteration τ is not more than the offer price of the prior round. The set of constraints (10) assures that the price paid by each bidder is bounded below by its VCG price and above by its maximum bid price (value).

Based on the prices generated in the above linear optimization problem, one calculates the *discount* for bidder j as the difference between the value of the winning bid and the price at this step of the algorithm. All bids of winning bidders are discounted by this amount and the winner determination problem ($WDP_{Core(\tau)}$) is again re-solved. The process continues until one can find no coalition that can increase revenue to the seller. This algorithm therefore finds prices for each winning bidder that are in the core. Since there may be many such minimum core prices, Day and Milgrom [18] suggest that one choose - in order to encourage sincere bidding - the minimum core prices that are closest in Euclidean distance from the VCG prices. Alternatively, Erdil and Klemperer [24] argue for a different set of minimum core prices that are based “on a class of ’reference rules’ in which bidders’ payments are, roughly speaking, determined independently of their own bids as far as possible”.

These core-selecting second-price sealed-bid mechanisms are relatively new. They have the following nice properties: They are in the core; they eliminate the exposure problem; and, they encourage bidders to bid sincerely. As with all sealed-bid auctions, they make collusion and punishment for not adhering to tacit agreements extremely difficult. In 2010, Portugal is scheduled to hold a core-selecting second-price auction (see announcement on Portuguese website anacom.pt).

There are, however, negatives associated with this auction design in that it puts a significant burden on the bidders. Each bidder needs to assess for each possible combination of items whether it is a package that it would like to acquire and then, for those that make sense for the bidder, provide a value for the package. In addition, such mechanisms do not provide any information about how the packages submitted might “fit” with packages submitted by other bidders. To overcome these problems, a number of authors have suggested new package-bidding designs that replicate many of the features of the simultaneous ascending auction.

5.2 Two Combinatorial Ascending Auction Designs

5.2.1 General Ascending Package-bidding Design:

In 2000, the FCC considered a package-bidding auction that resembled the SAA auction in many respects. It had most of the same rules as the SAA except that it allowed bidders to submit package bids. Each round a winner determination problem was solved to determine the winners with the constraint that at most one bid of any bidder could be in an optimal solution.

In an ascending auction, the auctioneer must announce prices for the next round. Since packages, rather than individual items may be winning, one needs to be able to allocate the package price among the individual elements that make up that package. If one has such individual item prices, then one can determine the price of any new package as the sum of the items that make up the package. However, since there is often a synergy value associated with the package, any such pricing approach cannot do this disaggregation so that there are minimal distortions from the true prices and so that prices do not fluctuate each round.

The reason for wanting such prices relates to a problem unique to package bidding auctions. The problem is labeled the *threshold problem*, whereby smaller bidders may have difficulty overcoming the high bid of a bid on a large package. An extreme example is when a bidder creates a package of *all* items and places a high price on that package bid. Each individual bidder need not make up the total shortfall between the collection of small bids and the large bidder, but rather they must *collectively* overcome the bid price and they need to know how much of the shortfall each should be paying. Essential to overcoming this threshold problem is providing good price information and having activity rules that force participation.

A number of pricing algorithms have been suggested that are based on the dual-price information that arises from solving the linear-programming relaxation to the integer programming problem. However, these prices are only approximations to the "true" dual prices of a combinatorial optimization problem, and are often call *pseudo-dual prices*. See Bikhchandani and Ostroy [5], Kwasnica et al. [36], Dunford et al. [21], and Bichler et al. [4] for more on pricing in combinatorial auctions settings.

The FCC combinatorial software uses the following pseudo-price estimates, although there are many others suggested in the literature. The pseudo-price estimates are obtained by solving the following optimization problem, Pseudo-Dual Price Calculation (PDPC):

$$[\text{PDPC}] \quad \min\, z_\delta^* = \sum_{j \in B \setminus W} \delta_i$$

subject to

$$\sum_{i \in I^j} \pi_i + \delta_j \geq b_j, \qquad \forall j \in B \setminus W, \tag{11}$$

$$\sum_{i \in I^j} \pi_i = b_j, \qquad \forall j \in W, \tag{12}$$

$$\delta_j \geq 0, \qquad \forall j \in B \setminus W, \tag{13}$$

$$\pi_i \geq r_i, \qquad \forall i \in I, \tag{14}$$

where j is the set of bids, I is the set of items, I^j is the set of all bids of bidder j, and r_i is the reserve price on item i. Constraint set (12) assures that the sum of the bid prices π_i of the items i in the winning bid is equal to the winning bid amount. Constraint set (11) together with the objective function tries to enforce dual feasibility as much as possible, where the δ_j's represent the deviation from dual feasibility[7]. Constraint set (13) maintains the non-negativity of these violations, and

[7] In *integer linear optimization*, there can be a duality gap whereby the primal and the dual problems do not provide the same values. In this case, if one uses the prices directly from the linear programming relaxation, one would obtain prices that would overestimate the prices needed. In

constraint set (14) forces the prices on all items to be at least the minimum price set by the auctioneer.

Since the solution to this problem is not necessarily unique, the algorithm tries — by solving a second optimization problem — to reduce the fluctuations in prices from round to round. To accomplish this, an optimization problem with a quadratic objective function and linear constraints is solved. The objective function that applies exponential smoothing to choose among alternative pseudo-dual prices with the additional constraint on the problem that the sum of the slack variables equals z^*_δ (the optimal value of the Pseudo-Dual Price Calculation). The objective function minimizes the sum of the squared distances of the resulting pseudo-dual prices in round t from their respective smoothed prices in round $t-1$. Thus, it is the pseudo-dual price of item i in round t. The smoothed price for item i in round t, is calculated using the following exponential smoothing formula: $p^t_i = \alpha^t_i + (1-\alpha)p^{t-1}_i$; where, p^{t-1}_i is the smoothed price in round $t-1$, $0 \le \alpha \le 1$, and p^0_i is the minimum opening bid amount for item i. The following quadratic program (QP) will find the pseudo-dual price, for each item i in round t that minimizes the sum of the squared distances from the respective smoothed prices in round $t-1$ while assuring that the pseudo-dual prices sum up to the provisionally winning bid amounts:

$$[\text{QP}] \quad \min \sum_{i \in L} (\pi^t_i - p^{t-1}_i)^2$$

subject to

$$\sum_{i \in I^j} \pi^t_i + \delta_j \ge b_j, \qquad \forall j \in B^i \backslash W^t, \tag{15}$$

$$\sum_{i \in I^j} \pi^t_i = b_j, \qquad \forall j \in W^t, \tag{16}$$

$$\sum_{j \in B^i \backslash W^t} \delta_j = z^*_\delta, \tag{17}$$

$$\delta_j \ge 0, \qquad \forall j \in B^i \backslash W^t, \tag{18}$$

$$\pi^t_i \ge r_i, \qquad \forall i \in I. \tag{19}$$

Note that problem (QP) has the same constraints as [PDPC], but has added the additional restriction (17) that the sum of the δ_j's is fixed to the value z^*_δ, the optimal value from [PDPC].

Experimental testing by Kwasnica et al. [36] has shown — when bidders have complementary values for packages — that package-bidding auctions that use pseudo-dual prices result in more efficient outcomes. However, laboratory experiments also found negatives to such designs. In testing by Goeree et al. [26] and Brunner et al. [8], results showed that such auctions took longer to complete and bidders found it difficult to construct packages that fit well with the packages of

the context of this application, overestimating prices could lead to inefficient outcomes. Thus, we make approximations that assure that the prices are consistent with the revenue obtained from the integer optimization.

other bidders. In addition, bidders were uncomfortable with the fact that prices fluctuated throughout the auction, see Dunford et al. [21] for more about the price fluctuations using pseudo-dual prices. The complexity of constructing bids increased with the number of items. Similarly, the computations required by the auctioneer also increase. Scalability of an auction design is an issue for a regulator that wants to use the same auction design for multiple auctions. For this reason, Goeree et al. [26] suggested that the FCC use the less complex hierarchical design for a single block of the 700 MHz auction. Here there were only two choices, the bidder either bids on all of the states as a single (nationwide) license, or one bids on any of the eight regional licenses. If the sum of the eight regional auction bids eclipses the top bid for the national license, the spectrum will be sold in those pieces. Otherwise, it will be sold as one nationwide license. This hierarchical approach predetermines the composition of the packages and through that packaging makes computation easy but does not provide the general flexibility of the more complex package-bidding designs.

5.2.2 Clock Designs:

In 2003, Porter et al. [45] studied a *combinatorial clock design* in laboratory settings and found that it yielded high efficiency and was better liked by the participants then their package-bidding ascending auction design. One can think of a clock design as similar to the Dutch Auction in which there is a clock for identical items and at every clock "tick" a price for each item is announced and each bidder has to agree to the new (higher) price for that item by continuing to bid on it. Once the price gets too high, the bidder acknowledges that he is no longer interested in that item. In a combinatorial setting, Porter et al. suggested a clock for each item in the auction. This auction is similar to the ascending non-package bidding design but allows bidders to express their bids as "all or nothing package bids". There is no concept of a "provisionally winning bidder" and a winner determination problem is not solved to determine if the packages fit together. Instead, prices increase on any item for which there is more demand than supply, i.e. for which there is more than one bidder bidding on that item. Since there are no provisional winners, in every round all bidders must rebid on any of the items that they wish to procure. The only information provided to bidders at the end of each round is the quantity demanded for each item and the price for the next round. At the end of a round, the auctioneer examines if there is excess demand on any item. If there is no excess demand, then this stage of the auction ends and those that bid in the last round win exactly what they bid.

It can happen that — because there are no provisionally winning bids — an item that had excess demand in the prior round now has excess supply since more than one bidder chose to no longer bid on that item. If at the end of the auction, there are items for which there is excess supply, Porter et al. [45] collect all bids submitted during the auction, treat all bids of a given bidder as mutually exclusive, and solve the winner determination problem. If the solution to the winner determination problem has supply exactly equaling demand, then the auction ends and the winners pay the prices bid. If not, the authors solve for pseudo-dual prices based on the winner determination problem outcome and restart the clock at these new (lower) prices. The process continues until supply exactly equals demand. This novel clock design was tested in the laboratory and found to have improved efficiency over the more general package-bidding designs and over the non-package SAA design. Bidders liked the design, because it provided price discovery, bidders only had to submit a single bid each round and prices were monotonically increasing throughout a clock stage. To our knowledge, this auction design has not been used for a spectrum auction. We present it here because it is similar to the combinatorial auction design described below which has been used for spectrum allocation.

Ausubel, Cramton and Milgrom [1] suggest a similar clock design, which they called the *clock-proxy design*. During the clock phase of the auction, the process is very similar to the Porter et al. [45] design except that they generalize the design to fungible items. That is, items that are nearly identical are treated as identical during this phase of the auction and grouped together. Items that are fungible have a single clock, and there may be a number of different clocks for different items (e.g. bandwidth may be fungible but regions are not). For each clock, bidders specify the quantity (i.e. the amount of bandwidth) they desire at a given price. Prices increase whenever demand is greater than supply. All bidders must rebid on any item that they wish to procure in each round. The only information provided to bidders at the end of each round is the quantity demanded for each item and the price for the next round. They suggest an activity rule that is based on a "price times quantity" basis, referred to as the *revealed-preference activity rule*. This rule removes some parking strategies[8] that were observed in the strictly quantity-based (MHz-Pop) rule.

Another novel aspect of this design is that it allows bidders who find the increment set by the auctioneer too steep to be able to specify a price somewhere between the old price and the new price that indicates the maximum amount the bidder is willing to pay for that combination of items. In this way, the efficiency loss due to increment size is lessened. The clock stage ends when demand is less than or equal to supply on all items. In some cases demand could have dropped to below supply because more than one bidder dropped quantity simultaneously.

To rectify this problem, and to allow the specification of bids on packages that had similar values to the ones specified in a given round of the clock auction, bidders are allowed to submit additional bids so long as each of the bids satisfy the activity rules. These new bids as well as all of the bids provided in the clock stage are then the bids considered within a second-price sealed-bid second auction where all bids of a given bidder are treated as mutually exclusive. Winners are determined by solving the winner determination problem and the amount paid is the second price minimal core outcome as described in Day and Milgrom [18]. Klemperer [34] suggested a similar (non-package) approach to spectrum allocation with an ascending auction followed by a final first-price sealed-bid auction.

[8] Parking is bidding on an item, usually early in the auction, that you do not want but that has sufficient demand so that one can satisfy one's activity requirements without bidding up the licenses that one wants.

In 2007, Ofcom[9] in the United Kingdom ran its first combinatorial clock auction. This auction had virtually all of the features of the Ausubel, Cramton, Milgrom [1] clock-proxy auction design except that the activity rule was a quantity-based rule. Ofcom had its second combinatorial clock auction in 2008. Early in 2010, Ofcom will hold its third auction using this design. The upcoming auction is for valuable spectrum in the 2010 to 2690 MHz range. A novel feature of this upcoming auction is that the regulators did not have to decide on a band plan prior to the auction nor did they decide how much of the spectrum would be paired or unpaired. Instead of dividing up the bands into spectrum widths prior to the auction, bidders in the combinatorial clock auction specify the quantity of spectrum they desire (in terms of number of 5MHz blocks) and whether they want paired or unpaired spectrum[10]. At the end of the auction, based on the preferences of the winning bidders, the spectrum is allocated to assure that the bidders receive the bandwidth needed, is consistent with their paired and unpaired specifications and assures that their spectrum is contiguous. A second auction, known as the *allocation auction*, allows winning bidders to specify *precisely* the location of their spectrum holdings. This second auction allows these winning bidders to bid the amount they are willing to pay (in addition to the bid price paid in the prior auction) for a given location on the band plan. In this way, the auctioneer does not need to limit possible configurations and the bidders are able to express their needs through their bids.

Cramton examines the results of the first two Ofcom combinatorial clock auctions: the $10-40$ GHz auction that took place in February 2009 [13] and the L-band auction that took place in May 2008 [14]. He argues that both ended in efficient outcomes. However, the data presented shows that the final prices obtained in the sealed-bid component of each auction were far less than that obtained in the clock phase. Specifically, in the $10-40$ GHz auction, there were 10 bidders and each won some package in that auction. However, the revenue from the final clock prices was roughly *five times higher* than the prices actually paid at the end of the second-price sealed-bid phase. Specifically, the clock phase ended with total revenue of 6,836,000 British pounds, whereas the final amount paid by all bidders was $1,417,500$ pounds. One likely reason for the disparity in prices between the two phases of the auction is that only two of the eight bidders provided a wide range of supplementary bids in the sealed-bid phase (BT submitted 545 bids and T-mobile submitted 107 bids). All other bidders submitted less than 23 bids each, with five submitting less than five bids. Since the bidders were not provided with information about the package forms of the bids submitted during the clock phase, it was difficult for bidders to deduce what additional bids would fit with the existing bids from the clock phase.

In the L-band Auction, there were 17 unique items up for lease. Eight bidders participated in the auction. t the end of the clock phase, Qualcomm was the high

[9] Ofcom is the independent regulator and competition authority for the UK communications industries, with responsibilities across television, radio, telecommunications and wireless communications services (Ofcom: http://www.ofcom.org.uk/)

[10] Wireless technologies differ and some technologies require that there be an uplink and a downlink separated by a certain width (e.g. LTE) while other technology (e.g. WIMAX) that does not require this pairing.

bidder on all 17 lots and remained the high bidder after the sealed-bid phase. In this case, Qualcomm bid 20 million pounds in the final bid phase (slightly higher than its final clock bid of 18.6 million pounds) but paid only 8.33 million pounds. Again, the reason for Qualcomm receiving these licenses for *less than half* of what was needed to overcome the bidders in the clock phase was that the bids submitted in the sealed-bid phase did not fit well together. Indeed, the second-highest coalition (i.e. the coalition that determined what Qualcomm paid) consisted of a set of bids that did not cover six of the 17 licenses. Thus, Qualcomm paid nothing for these six licenses.

As Ausubel, Cramton and Milgrom [1] state, the purpose of the clock stage is for price discovery. The results of the only two auctions that have used this design indicate that the price signals provided in this stage significantly overestimate the prices actually paid at the end of the auction. Since the design does not allow bidders to understand how synergistic valuations are impacting the prices presented, it is hard for bidders to understand the common-value component of the bids expressed in the clock phase. Also, as Cramton [14] points out, the activity rule between the two phases might have hindered their ability to submit supplementary package bids. More likely, the bidders did not understand how submitting more bids could impact final prices. In the L-band auction, all bidders understood that Qualcomm had already won all licenses. They knew that if Qualcomm bid only one increment above its winning price in the clock phase that it was guaranteed to win all licenses in the sealed-bid phase. It is not clear that they understood that they could have significantly impacted how much Qualcomm paid by submitting supplemental bids.

More testing needs to be done to better understand what should be done when there is excess demand. Should one have a final-round sealed-bid phase? Would it be better to use the intra-round bidding to eliminate the excess demand, or — as Porter et al. [45] suggest — should one restart the clock after solving the winner determination problem with the packages submitted in the clock phase? If a final-round sealed-bid phase takes place, how much information do bidders need to be able to overcome the fitting and threshold problems?

Most countries still use the (non-package) SAA auction design although many are considering changing to some form of the combinatorial clock design. The clock design is especially useful when the licenses being offered are for different amounts of fungible bandwidth, and each license is a nationwide license. For such applications, very few clocks are needed and bidders treat bands of spectrum as fungible. In addition to the United Kingdom, this auction design has been used in Trinidad and Tobago. It is also being considered for use in upcoming auctions in The Netherlands, Denmark and Austria.

In the US and Canada, it is not clear how to design a clock-proxy auction when the licenses are for multiple small regions. In this case, the proxy stage of allowing multiple package bids may be difficult for bidders as there may be an enormous number of new bids that they may wish to submit. And, in order to keep the optimization problem tractable, the auctioneer may have to limit the number of supplemental bids any bidder can submit. The impacts on efficiency of such rules are unclear. Thus, there is significant uncertainty as to where this auction design might work best and where some other package-bidding design might be better.

Clearly, combinatorial auctions are more complex that the SAA or simpler clock designs. When Cramton [15] tested the combinatorial clock-proxy design, he used Ph.D. students who had taken advanced courses in game theory and auction theory. He discovered that even these bidders found the revealed-preference rule too complex especially as it related to composing new supplementary bids for the sealed-bid phase of the auction. Cramton suggests that bidders need help and suggested that the bidding system could provide a bidder with a set of bids that satisfy all constraints (both revealed preference and budget constraints). In addition, the system could provide additional bids that are closest to the bids a bidder wanted to submit but that violated the revealed-preference constraints. His experiments also showed that the revealed preference rule may, at times, be too restrictive either because bidders are faced with budget constraints or because bidders change their values over the course of the auction as a result of learning the common value component of a given item. We discuss further the need for bidder-aide tools in the next section.

6 The Need for Bidder-aide Tools

Since multi-item auctions are complex and require bidders to consider multiple alternative bid options, we believe that it important that the computer software associated with such auctions be easy to use and understand. Good graphical user interfaces help bidders to feel comfortable that they understand the current state of the auction (they have been able to find the current price information, the amount of bidding necessary to remain eligible, their dollar exposure based on what they have bid, the degree of demand for different items, etc.). The system must also provide easy ways for them to input their next bids and confirm that they have provided the system with the correct information. Computer interfaces for such processes continue to improve and to provide better ways of displaying information to the users through charts, graphs and pictures. We foresee continued improvement in this area.

These tools do not, however, help the bidder determine the optimal combination of items to bundle as a package and the optimal number of packages to supply to the system. Although all of us face similar decisions regularly — we are bombarded with a near limitless variety of purchasing opportunities for even the simplest of tasks, e.g., what to eat for dinner — when the stakes are high and the number of substitutes is limited, the problem of providing decision support tools becomes more pressing. Thus, there is a need for the auction community to create tools that will help narrow the scope of the search and suggest good packages based on the bidder-specific business plans and preferences.

As the number of items in an auction grows, there is an exponential growth in the effort related to bid preparation and communication. Optimization tools that translate business plans into profitable packages will lessen the bidder's burden. We refer the reader to a few papers that have begun work in this very important area. For some discussion on this topic, see Hoffman [28] for tools specific to the spectrum auction setting, and Boutelier [7] and Elmaghraby and Keskinocak [22] for tools for other applications. Also, Day and Raghavan [19] and Parkes [44] provide alternative ways of bidders expressing preferences that preclude the bidder specifying specific packages to the auctioneer.

In spectrum auctions, bidders usually think of valuing spectrum based on "MHz Pop" calculations (i.e. the population within the region times the amount of spectrum). A bidder-aide tool for this application might ask bidders to supply information in terms of their business plans: What geographic areas are their primary, secondary and tertiary markets? What is their minimum and maximum bandwidth needs? Do they prefer certain bands? How much population do they need to have a viable business? How much are they willing to spend per MHz population unit? Are their exceptions for certain important regions? With this information, the tool translates their needs into constraints for an optimization model that maximizes the bidder's profit, given the current purchase prices for the licenses of interest. Packages are then provided back to the user for evaluation. The process continues with the bidder having the ability to change his business plan, budget or detailed specifications within a plan. The tool re-optimizes and creates new packages. Once the bidder is satisfied with the outputs, he indicates the package or packages (depending on the auction design) that are best, and these are supplied to the auctioneer. Tools similar in nature are being used by the major supply-chain auction providers. They must always be designed for the specific application and include all of the constraints relevant to that business concern. However, with such tools, more entities are willing to participate in the auction thereby improving the overall competitiveness of the auction.

7 Conclusions

Auctions have proven quite successful in allocating spectrum. Auctions have brought greater transparency to the overall process and have allowed the spectrum to be allocated in a timely manner. Recently, auction theorists have been arguing for more complicated auctions that allow the packaging of licenses. One reason for this new interest is the ability of current software — based on sophisticated mathematical algorithms developed over the past 20 years — to be able to solve very large combinatorial optimization problems routinely. In addition, the game theorists and the experimental economists have highlighted issues with prior designs and provided new mechanisms that have nice theoretical properties. Because of the need for sophisticated allocation mechanisms, computer scientists, operations research analysts, experimental economists and game theorists are working together to determine new approaches for the allocation of these resources.

As broadband applications multiply, the wireless telecommunications industry clamors for more spectrum to satisfy the seemingly insatiable appetite for applications that require more and more bandwidth. Research into new auction designs, new bidding tools, new bidding languages and the testing of such approaches in laboratory settings makes this a growing area to study. Since much of the radio spectrum is already allocated, it is likely that spectrum exchanges that encourage the reuse of existing spectrum more efficiently and more flexibly will also be a growing area of research.

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