

OR 664 / SYST 664 / CSI 674: Homework Assignment 4

due February 20, 2023, 11:59PM

Please make sure you mark clearly which question you are answering and that you explain how you arrived at your answer. Your response will be graded for correctness and clarity. Points may be deducted if you do not provide information on how you arrived at your answer. Upload your responses to Gradescope. Please submit R code either as a separate attachment on Blackboard or in your main submission.

1. In previous years, students in this course collected data on people's preferences in the two Allais gambles from Assignment 2. For this problem, we will assume that responses are independent and identically distributed, and the probability is θ that a person chooses both B in the first gamble and C in the second gamble.
 - a. Assume that the prior distribution for θ is Beta(1, 3). Find the prior mean and standard deviation for θ . Find a 95% symmetric tail area credible interval for the prior probability that a person would choose B and C. Do you think this is a reasonable prior distribution to use for this problem? Why or why not?
 - b. In 2009, 19 out of 47 respondents chose B and C. Find the posterior distribution for the probability θ that a person in this population would choose B and C. Name the type of distribution and the posterior hyperparameters.
 - c. Find the posterior mean and standard deviation. Find a 95% symmetric tail area credible interval for θ .
 - d. Make a triplot of the prior distribution, normalized likelihood and posterior distribution.
 - e. Comment on your results.

2. This problem continues analysis of the vehicle traffic data from Assignment 3. As in Assignment 3, assume that counts of vehicles per 15-second interval are independent and identically distributed Poisson random variables with unknown mean Λ .
 - a. Assume that Λ , the rate parameter of the Poisson distribution for counts, has a continuous gamma prior distribution for Λ with shape 1 and scale 10^6 . (The gamma distribution with shape 1 tends to a uniform distribution as the scale tends to ∞ , so this prior distribution is "almost" uniform.) Find the posterior distribution of Λ . State the distribution type and hyperparameters.
 - b. Find the posterior mean and standard deviation of Λ . Compare your results to Problem 2 of Assignment 3. Discuss.
 - c. Find a 95% symmetric tail area posterior credible interval for Λ . Find a 95% symmetric tail area posterior credible interval for Θ , the mean time between vehicle arrivals.