

# OR 664 / SYST 664 / CSI 674: Homework Assignment 5

due February 27, 2023, 11:59PM

Please mark clearly which question you are answering and that you explain how you arrived at your answer. Your response will be graded for correctness and clarity. Points may be deducted if you do not provide information on how you arrived at your answer. Upload your responses to Gradescope. Please submit R code either as a separate attachment on Blackboard or in your main submission.

1. Use the posterior distribution from Problem 2 of Assignment 4 to find the predictive distribution for the number of cars passing in the next minute. Name the family of distributions and the parameters of the predictive distribution. Find the mean and standard deviation of the predictive distribution. Find the probability that more than 10 cars will pass in the next minute. (*Hint: one minute is four 15-second time intervals.*)
2. Chronic obstructive pulmonary disease (COPD) is a common lung disease characterized by difficulty in breathing. A substantial proportion of COPD patients admitted to emergency medical facilities are released as outpatients. A randomized, double-blind, placebo-controlled study examined the incidence of relapse in COPD patients released as outpatients as a function of whether the patients received treatment with corticosteroids.<sup>1</sup> A total of 147 patients were enrolled in the study and were randomly assigned to treatment or placebo group on discharge from an emergency facility. Seven patients were lost from the study prior to follow-up. For the remaining 140 patients, the table below summarizes the primary outcome of the study, relapse within 30 days of discharge.

	Relapse	No Relapse	Total
Treatment	19	51	70
Placebo	30	40	70
Total	49	91	140

- a. Let  $Y_1$  and  $Y_2$  be the number of patients who relapse in the treatment and placebo groups, respectively. Assume  $Y_1$  and  $Y_2$  are independent Binomial( $70, \theta_i$ ) distributions, for  $i=1,2$ . Assume  $\theta_1$  and  $\theta_2$  have independent Beta prior distributions with shape parameters  $\frac{1}{2}$  and  $\frac{1}{2}$  (this is the Jeffreys prior distribution). Find the joint posterior distribution for  $\theta_1$  and  $\theta_2$ . Name the distribution type and its hyperparameters.
  - b. Generate 5000 random pairs  $(\theta_{1k}, \theta_{k2})$ ,  $k=1, \dots, 5000$  from the joint posterior distribution for  $(\theta_1, \theta_2)$ . Use this random sample to estimate the posterior probability that the rate of relapse is lower for treatment than for placebo. Discuss your results.
3. A web designer is analyzing traffic on a web site. Assume the number of visitors arriving at the site at a given time of day is modeled as a Poisson random variable with a rate of  $\Lambda$  visitors per minute. Based on prior experience with similar web sites, the following estimates are given:
    - There is a 90% probability that the rate is greater than 5 visitors per minute.
    - The rate is equally likely to be greater than or less than 14 visitors per minute.
    - There is a 90% probability that the rate is less than 27 visitors per minute.

Find a Gamma prior distribution for the arrival rate that fits these judgments as well as possible. Comment on your results. (*Hint: there is no "right" answer to this problem. You can use trial and error to find a distribution that fits as well as possible. You can also use an optimization method such as Excel Solver to minimize a measure of how far apart the given quantiles are from the ones in the target distribution.*)

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<sup>1</sup> S. D. Aaron, K. L. Vandemheen, P. Hebert, R. Dales, I. G. Stiell, J. Ahuja, G. Dickinson, R. Brison, B. H. Rowe, J. Dreyer, E. Yetisir, D. Cass, and G. Wells, "Outpatient Oral Prednisone after Emergency Treatment of Chronic Obstructive Pulmonary Disease," *New England Journal of Medicine*, vol. 348, no. 26, pp. 2618–2625, 2003.