

SYST/STAT 664: Homework Assignment 10

due April 26, 2022

Please make sure you mark clearly which question you are answering and that you explain how you arrived at your answer. Your response will be graded for correctness and clarity. Points may be deducted if you do not provide information on how you arrived at your answer. Upload your responses to Gradescope. Please submit R code either as a separate attachment on Blackboard or in your main submission.

1. A compound called estriol was measured over a 24 hour period in the blood of pregnant women. The babies' weights were then recorded at birth. The purpose of the study was to determine whether there was a relationship between estriol and birth weight.¹ The data from this study can be found at <http://www.biostat.umn.edu/~lynn/iid/estriol.dat>. Estriol was measured in milligrams per 24 hours and birthweight was measured in grams/100.

Assume the relationship between birthweight $y_{1:n}$ and estriol $x_{1:n}$ is linear with independent normally distributed errors. Assume the following unit information prior distribution for the parameters: (1) Conditional on the precision ρ , the transformed intercept η is normally distributed with mean 35 (the average birth weight of a baby is about 3500 g) and precision ρ ; (2) Conditional on the precision ρ , the slope β is independent of η and normally distributed with mean 0 and precision $\frac{1}{n}S_{xx}\rho$; (3) the precision ρ has a gamma distribution with shape $\frac{1}{2}$ and scale $2(n - 2)/S_{ee}$. Find the joint posterior distribution for the transformed intercept η , the slope β , and the precision ρ . Find 95% credible intervals for the slope β , the untransformed intercept α , and the standard deviation σ .

2. Comment on your results, including whether the assumptions for normal linear regression are met.
3. What is the predictive distribution for birthweight given milligrams of estriol? Find a 90% predictive interval for the birthweight of a baby given that 19 mg of estriol were measured in the mother's urine over 24 hours.
4. Use 1000 direct Monte Carlo samples to find an approximate 90% predictive interval for the birthweight of a baby given that 19 mg of estriol were measured in the mother's urine over 24 hours. Compare with Problem 3. Discuss.

¹ Original source: Greene and Touchstone (1963). 'Urinary tract estriol: an index of placental function,' American Journal of Obstetrics and Gynecology, 85:1-9. Reprinted in Rosner (1982). Fundamentals of biostatistics, Duxbury Press.