OR 664 / SYST 664 / CSI 674: Homework Assignment 2

due February 7, 2022, 11:59PM

Please make sure you mark clearly which question you are answering and that you explain how you arrived at your answer. Your response will be graded for correctness and clarity. Upload your responses to Gradescope.

1. Tarone (1982) reports data from 71 studies on tumor incidence in rats.[1]
    a. In one of the studies, 2 out of 13 rats had tumors. Assume there are 20 possible tumor probabilities: 0.025, 0.075, …, 0.975. Assume that the tumor probability is uniformly distributed. Find the posterior distribution for the tumor probability given the data for this study.
    b. Repeat Part a for a second study in which 1 in 18 rats had a tumor.
    c. Parts a and b assumed that each study had a different tumor probability, and that these tumor probabilities were uniformly distributed *a priori*. Now, assume the tumor probabilities are the same for the two studies, and that this probability has a uniform prior distribution. Find the posterior distribution for the common tumor probability given the combined results from the two studies.
    d. Compare the three distributions for Parts a, b, and c. Comment on your results.

2. In an experiment, subjects were given the choice between two gambles:

    Gamble 1:

    A: \$2500 with probability 0.33
    \$2400 with probability 0.66
    \$0 with probability 0.01

    B: \$2400 with certainty

    Suppose that a person is an expected utility maximizer. Set the utility scale so that u(\$0) = 0 and u(\$2500) = 1. Whether a utility maximizing person would choose Option A or Option B depends on the person's utility for \$2400. For what values of u(\$2400) would an expected utility maximizer choose Option A? For what values would an expected utility maximizer choose Option B?

    Gamble 2:

    C: \$2500 with probability 0.33
    \$0 with probability 0.67

    D: \$2400 with probability 0.34
    \$0 with probability 0.66

    For what values of u(\$2400) would an expected utility maximizer choose Option C? For what values would an expected utility maximizer choose Option D?

    This problem is a version of the famous *Allais paradox*, named after the prominent critic of subjective expected utility theory who first presented it. Kahneman and Tversky[2] found that 82% of subjects preferred B over A, and 83% preferred C over D. Explain why no expected utility maximizer would prefer *both* B in Gamble 1 and C in Gamble 2. (*A utility maximizer might prefer B in Gamble 1. A different utility maximizer might prefer C in Gamble 2. But the same utility maximizer would not prefer both B in Gamble 1 and C in Gamble 2.*) Discuss these results. Why do you think many people prefer B in Gamble 1 and C in Gamble 2? Do you think this is reasonable even if it does not conform to expected utility theory?

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[1] Tarone, R. E. (1982) The Use of Historical Control Information in Testing for a Trend in Proportions. *Biometrics* **38**, 215-220.

[2] Kahneman, D., P. Slovic, et al. (1982). *Judgment under Uncertainty: Heuristics and Biases*. Cambridge University Press.