OR 664 / SYST 664 / CSI 674: Homework Assignment 4
due February 21, 2022, 11:59PM

Please make sure you mark clearly which question you are answering and that you explain how you arrived at your answer. Your response will be graded for correctness and clarity. Points may be deducted if you do not provide information on how you arrived at your answer. Upload your responses to Gradescope. Please submit R code either as a separate attachment on Blackboard or in your main submission.

1. In previous years, students in this course collected data on people's preferences in the two Allais gambles from Assignment 2. For this problem, we will assume that responses are independent and identically distributed, and the probability is θ that a person chooses both B in the first gamble and C in the second gamble.
    a. Assume that the prior distribution for θ is Beta$(1, 3)$. Find the prior mean and standard deviation for θ. Find a 95% symmetric tail area credible interval for the prior probability that a person would choose B and C. Do you think this is a reasonable prior distribution to use for this problem? Why or why not?
    b. In 2009, 19 out of 47 respondents chose B and C. Find the posterior distribution for the probability θ that a person in this population would choose B and C. Name the type of distribution and the posterior hyperparameters.
    c. Find the posterior mean and standard deviation. Find a 95% symmetric tail area credible interval for θ.
    d. Make a triplot of the prior distribution, normalized likelihood and posterior distribution.
    e. Comment on your results.

2. This problem continues analysis of the automobile traffic data from Assignment 3. As in Assignment 3, assume that counts of cars per 15-second interval are independent and identically distributed Poisson random variables with unknown mean Λ. Assume your knowledge of Λ is expressed by the posterior distribution from Problem 2c of Assignment 3. (Solutions to Assignment 3 will be posted by Wednesday and will be visible if you have turned in Assignment 3.)
    a. Find the predictive distribution for the number of cars passing in the next 15-second interval. Name the family of distributions and the parameters of the predictive distribution. Find the predictive probability that 0, 1, 2, 3, 4, and more than 4 cars will pass the point in the next 15 seconds.
    b. Find the predictive distribution for the number of cars passing in the next minute. Name the family of distributions and the parameters of the predictive distribution. Find the mean and standard deviation of the predictive distribution. Find the probability that more than 10 cars will pass in the next minute.