SYST/STAT 664: Homework Assignment 6

due March 28, 2022

Please make sure you mark clearly which question you are answering and that you explain how you arrived at your answer. Your response will be graded for correctness and clarity. Points may be deducted if you do not provide information on how you arrived at your answer. Upload your responses to Gradescope. Please submit R code either as a separate attachment on Blackboard or in your main submission.

1. Concentrations of the pollutants aldrin and hexachlorobenzene (HCB) in nanograms per liter were measured in ten surface water samples, ten mid-depth water samples, and ten bottom samples from the Wolf River in Tennessee. The samples were taken downstream from an abandoned dump site previously used by the pesticide industry. The full data set can be found at http://www.biostat.umn.edu/~lynn/iid/wolf.river.dat. For this problem, we consider only HCB measurements taken at the bottom and the surface. The question of interest is whether the distribution of HCB concentration depends on the depth at which the measurement was taken. The data for this problem are given below.

Surface	Bottom
3.74	5.44
4.61	6.88
4.00	5.37
4.67	5.44
4.87	5.03
5.12	6.48
4.52	3.89
5.29	5.85
5.74	6.85
5.48	7.16

Assume the observations are independent normal random variables with unknown depth-specific means Θ_s and Θ_b and precisions P_s and P_b. Assume independent improper reference priors for the surface and bottom parameters:

$$g(\theta_s, \theta_b, \rho_s, \rho_b) = g(\theta_s, \rho_s)\, g(\theta_b, \rho_b) \propto \rho_s^{-1}\rho_b^{-1}.$$

This prior can be treated as the product of two normal-gamma priors with $\mu_s = \mu_b = 0$, $k_s = k_b = 0$, $\alpha_s = \alpha_b = -1/2$, and $\beta_s = \beta_b = \infty$. (These are not valid normal-gamma distributions, but you can use the usual Bayesian conjugate updating rule to find the posterior distribution.) Find the joint posterior distribution for the parameters $(\theta_s, \theta_b, \rho_s, \rho_b)$. State the type of distribution and the posterior hyperparameters. Find 90% posterior credible intervals for $\theta_s, \theta_b, \rho_s$ and ρ_b. Comment on your results.

2. Use direct Monte Carlo to sample 10,000 observations from the joint posterior distribution of $(\theta_s, \theta_b, \rho_s, \rho_b)$. Use your Monte Carlo samples to estimate 90% posterior credible intervals for $\theta_s, \theta_b, \rho_s$ and ρ_b. Compare with the result of Problem 1.

3. Use your direct Monte Carlo sample to estimate the probability that the mean bottom concentration θ_b is higher than the mean surface concentration θ_s and to estimate the probability that the standard deviation σ_b of the bottom concentrations is higher than the standard deviation σ_s of the surface concentrations.

4. Comment on your analysis. What are your conclusions about the distributions of surface and bottom concentrations? Is the assumption of normality reasonable? Are the means different for surface and bottom? The standard deviations?