SYST/STAT 664: Homework Assignment 7

due April 4, 2022

Please make sure you mark clearly which question you are answering and that you explain how you arrived at your answer. Your response will be graded for correctness and clarity. Points may be deducted if you do not provide information on how you arrived at your answer. Upload your responses to Gradescope. Please submit R code either as a separate attachment on Blackboard or in your main submission.

1. This assignment continues with the pollution data from Assignment 6: HCB measurements taken at the bottom and the surface from Wolf River in Tennessee:

Surface	Bottom
3.74	5.44
4.61	6.88
4.00	5.37
4.67	5.44
4.87	5.03
5.12	6.48
4.52	3.89
5.29	5.85
5.74	6.85
5.48	7.16

As with Assignment 6, assume the observations are normally distributed with unknown depth-specific means Θ_s and Θ_b and precisions P_s and P_b. Assume that experts have provided the following prior information based on previous studies.

- The unknown means Θ_s and Θ_b are independent and normally distributed with mean μ and standard deviation τ. The unknown precisions P_s and P_b are independent of Θ_s and Θ_b and have gamma distributions with shape α and scale β.
- Experts specified a 95% prior credible interval of [3, 9] for Θ_s and Θ_b. A good fit to this credible interval is obtained by setting the prior mean to μ =6 and the prior standard deviation to τ=1.5.
- A 95% prior credible interval of [0.75, 2.0] is given for the unknown standard deviations Σ_s and Σ_b. This translates to a credible interval of [0.25, 1.8] for $P_s = \Sigma_s^{-2}$ and $P_b = \Sigma_b^{-2}$. A good fit to this credible interval is obtained by setting the prior shape to α = 4.5 and the prior scale to β = 0.19.

1. Find the following conditional distributions. The parameters of these distributions will be formulas *containing other parameter(s).*
    - The conditional distribution for Θ_s given the other parameters and the observations.
    - The conditional distribution for Θ_b given the other parameters and the observations.
    - The conditional distribution for P_s given the other parameters and the observations.
    - The conditional distribution for P_b given the other parameters and the observations.
2. Using the distributions you found in Part 1, draw 10,000 Gibbs samples of (Θ_s, Θ_b, P_s, P_b). Estimate 90% credible intervals for Θ_s, Θ_b, $\Sigma_s = P_s^{-1/2}$, $\Sigma_b = P_b^{-1/2}$, and Θ_b - Θ_s.
3. Do a traceplot of Θ_b - Θ_s. Find the autocorrelation function of Θ_b - Θ_s and the effective sample size for your Monte Carlo sample for Θ_b - Θ_s.
4. Comment on your results. Compare with Assignment 6.