

SYST/STAT 664: Homework Assignment 8

due April 11, 2022

Please make sure you mark clearly which question you are answering and that you explain how you arrived at your answer. Your response will be graded for correctness and clarity. Points may be deducted if you do not provide information on how you arrived at your answer. Upload your responses to Gradescope. Please submit R code either as a separate attachment on Blackboard or in your main submission.

1. A biologist counts the number of sparrows visiting six bird feeders placed on a given day.

Feeder	Number of Birds
1	11
2	22
3	13
4	24
5	19
6	16

- Assume that the bird counts are independent Poisson random variables with feeder-dependent means λ_i , for $i=1, \dots, 6$.
- Assume that the means λ_i are independent and identically distributed gamma random variables with shape α and scale β (or equivalently, shape α and mean $m = \alpha\beta$)
- The mean $m = \alpha\beta$ of the gamma distribution is uniformly distributed on a grid of 200 equally spaced values starting at 5 and ending at 40.
- The shape α is independent of the mean m and has a distribution that takes values on a grid of 200 equally spaced points starting at 1 and ending at 50, with prior probabilities proportional to a gamma density with shape 1 and scale 5.

Use Gibbs sampling to draw 10000 samples from the joint posterior distribution of the mean m , the shape parameter α , and the six mean parameters λ_i , $i=1, \dots, 6$, conditional on the observed bird counts. Using your sample, calculate 95% credible intervals for the mean m , the shape α , and the six mean parameters λ_i , $i=1, \dots, 6$.

2. Find the effective sample size for the Monte Carlo samples of the mean m , the shape parameter α , and the six mean parameters λ_i , $i=1, \dots, 6$.
3. Do traceplots for the mean m , the shape parameter α , and the six rate parameters λ_i , $i=1, \dots, 6$.
4. The fourth feeder had the highest bird count and the first feeder had the lowest bird count. Use your Monte Carlo sample to estimate the posterior probability that the first feeder has a smaller mean bird count than the fourth feeder. Explain how you obtained your estimate.
5. Discuss your results.