SYST/STAT 664: Homework Assignment 9

due April 19, 2022

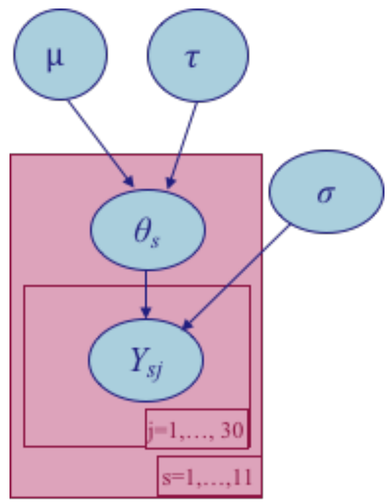
Please make sure you mark clearly which question you are answering and that you explain how you arrived at your answer. Your response will be graded for correctness and clarity. Points may be deducted if you do not provide information on how you arrived at your answer. Upload your responses to Gradescope. Please submit R code either as a separate attachment on Blackboard or in your main submission.

1. Section 18.4 of Gelman, et al. (2013) analyzes data on reaction times for 11 non-schizophrenic and 6 schizophrenic subjects. The data set can be found at:

   http://www.stat.columbia.edu/~gelman/book/data/schiz.asc

   The first 11 rows are data for the non-schizophrenic subjects. (We analyzed data from one of these subjects in Unit 5.) Gelman, et al. assume that the natural logarithms of the response times for each non-schizophrenic subject are independent normal random variables with person-specific mean θ_j (j = 1,…11) and common variance σ^2. Discuss whether you think this assumption is reasonable.

2. Consider a hierarchical model for the natural logarithms of reaction times of the non-schizophrenic subjects. A plate diagram for the model is shown at the right.



   a. The natural logarithms Y_{sj} of the response times are independent normal random variables with person-specific mean θ_s (s = 1,…11).
   b. All the observations have the same standard deviation σ.
   c. The 11 means θ_s, s=1, …, 11, are independent and identically distributed normal random variables with mean μ and standard deviation τ.
   d. The parameters σ, μ and τ are independent of each other.
   e. Assume an empirical Bayes model for the top-level parameters. Use the following point estimates for σ, μ and τ.
   - Estimate σ by the average of the sample standard deviations of the log reaction times of the 11 subjects.
   - Estimate the prior mean μ by the grand mean of all the log reaction times.
   - Estimate the standard deviation τ by the standard deviation of the eleven sample means $\bar{\bar{x}}_s$, s=1, …, 11.

   Using these empirical Bayes estimates of σ, μ and τ, find the eleven posterior distributions for θ_s, s=1, …, 11.

3. Find 95% credible intervals for each of the eleven means θ_j (j = 1,…11). Discuss your results, especially whether and how the distribution of reaction times differs across subjects.